Common Core • Mathematics

DOCUMENTS REVIEWED

Common Core State Standards for Mathematics. June 2, 2010.
Accessed from: http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Overview

The final version of the Common Core State Standards for math is exemplary in many ways. The expectations are generally well written and presented, and cover much mathematical content with both depth and rigor. But, though the content is generally sound, the standards are not particularly easy to read, and require careful attention on the part of the reader.

GRADE A-		
	Clarity and Specificity:	2/3
	Content and Rigor:	7/7
	Total Score:	**9/10**

The development of arithmetic in elementary school is a primary focus of these standards and that content is thoroughly covered. The often-difficult subject of fractions is developed rigorously, with clear and careful guidance. The high school content is often excellent, though the presentation is disjointed and mathematical coherence suffers. In addition, the geometry standards represent a significant departure from traditional axiomatic Euclidean geometry and no replacement foundation is established.

Despite some weaknesses, the Common Core standards provide a solid framework for learning rigorous mathematics.

General Organization

The K-8 standards are organized into grade-specific content "domains" such as "Numbers and Operations—Fractions" and "Expressions and Equations." The domains are further divided into grade-specific topic "clusters," and the grade-level standards are listed within these topic clusters. Each grade includes an overview that describes the most important content for that year.

The high school standards follow a slightly different structure. First, they are organized into five "conceptual categories," such as "functions" and "algebra." Each category comes with an introduction to the mathematics covered in that category and the list of topics. The standards are then presented by topic, and more advanced standards ("that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics") are given a special label.

Finally, the standards are introduced with a set of eight overarching "Standards for Mathematical Practice," which are basically process standards and are intended to be integrated into the teaching of mathematics at all levels.

Clarity and Specificity

With some exceptions, the K-8 standards are well organized. While many states apply one set of strands or topics to all grade levels, the Common Core varies the content domains and topic clusters from grade to grade, which results in relatively few extraneous or overly inflated standards.

Many standards are clear and specific. In addition, they make frequent and exemplary use of examples to clarify intent, such as:

> Tell and write time in hours and half-hours using analog and digital clocks (grade 1)
>
> Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure (grade 4)

> Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? (grade 6)

Though the standards are not succinct, which detracts from the ease of reading, careful reading reveals that they are generally both literate and mathematically correct—a rare combination in standards. The following excessively specific standard illustrates this:

> Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps (grade 1)

Unfortunately, despite the inclusion of examples, some standards are not specific enough to determine the intent, and they are subject to quite a bit of interpretation on the part of the reader. For example:

> Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time (grade 6)

This dense standard is difficult to follow, and the example does not provide enough guidance to help the reader understand what, precisely, students should know and be able to do.

The high school standards, in particular, are often too broadly stated to interpret. For example:

> Define appropriate quantities for the purpose of descriptive modeling (high school)
>
> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods (high school)
>
> Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant (high school)

The high school standards also manifest organizational problems. Grouping them into conceptual categories rather than by content artificially separates standards covering related topics. A clearer organizational structure would group such standards together in a mathematically coherent way.

The treatment of quadratics illustrates this problem. A complete and coherent analysis of quadratics provides students with experience with deep mathematics and exposure to many real-world applications, yet the basic analysis of quadratics is not placed in one coherent section. Instead, standards dealing with quadratics appear in three conceptual categories, and are even further separated by topic within the conceptual category of "algebra." An example of this is the following two closely related standards. The first is found under algebra, and the second under functions:

> Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form (algebra)
>
> Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context (functions)

This presentation is artificial; it would be improved by presenting these related standards together to reflect a rigorous development of theory and techniques.

The conceptual category of "functions" is particularly problematic. Ideally, linear functions and equations should be grouped together, and quadratic equations and functions should be grouped together. The Common Core, however, includes expectations that lump all of this content together. Take, for example, the following:

> Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
>
> a. Graph linear and quadratic functions and show intercepts, maxima, and minima (functions)

In this standard, linear and quadratic functions are inappropriately lumped together and then maxima and minima are asked for, and this only applies to quadratics.

Clarity and Specificity Conclusion

The K-8 Common Core standards are generally well organized and presented. An excellent feature is their use of examples to clarify intent. However, the standards are often long and difficult to read, and some of them are not clear. In addition, in high school, the presentation is not always coherent. The standards "do not quite provide a complete guide to users" and therefore receive a Clarity and Specificity score of two points out of three. (See *Common Grading Metric*, Appendix A.)

Content and Rigor

Content Priorities

Standards should clearly articulate what is most important for students to learn. Many existing standards, however, fail to explicitly set priorities for the content, which leaves the reader with no guidance about which standards are most important. This is unfortunate, particularly in the elementary grades, because the early development of arithmetic is the foundation for future mathematics and should be distinguished as the most important content. For example, crucial standards about learning to add should take priority over predicting the result of playing with dice (or spinners). Unfortunately, both of these are frequently mentioned in the early grades and, in the absence of any guidance, appear to have equal priority.

Common Core avoids this widespread problem. It sets excellent priorities that are expressed both explicitly and implicitly. The grade-level overviews for elementary school offer explicit guidance by identifying the three or four areas that students are expected to master in each grade and making it clear that arithmetic is the most important topic in the early grades. This is further supported by the standards themselves, of which well over half deal with arithmetic. This prioritization of arithmetic, which provides the foundation for the subsequent study of mathematics, is exemplary.

Content Strengths

The standards have many strong features and cover a lot of rich mathematics. The K-8 standards are well presented and not overly numerous. In particular, and in marked contrast to many existing state standards, they are not overwhelmed with extraneous standards in the early grades. In addition, they are generally mathematically sound, and the content is usually presented coherently.

Arithmetic is well covered. Instant recall of the number facts is required for addition and multiplication, though, as noted below, not for corresponding subtraction and division facts. The capstone standards for whole-number arithmetic are stated clearly and unambiguously:

> Fluently add and subtract multi-digit whole numbers using the standard algorithm (grade 4)
>
> Fluently multiply multi-digit whole numbers using the standard algorithm (grade 5)
>
> Fluently divide multi-digit numbers using the standard algorithm (grade 6)

Properties of the arithmetic operations are well developed and covered thoughtfully.

Fractions are developed rigorously and with a great deal of specificity. (In fact, the excellent guidance included here would improve the presentation of fractions in most textbooks.) The often-confused concept of fractions as numbers is introduced early and clearly, as demonstrated by the third-grade topic, "Developing an understanding of fractions as numbers." The arithmetic of fractions is carefully developed using mathematical reasoning. For example, part of the sequence is:

> Understand a fraction as a number on the number line; represent fractions on a number line diagram
>
> a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line
>
> b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line (grade 3)
>
> Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4) (grade 4)
>
> Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd) (grade 5)

Fractions are considered as division, and the standards include multiplying fractions by whole numbers, and then by fractions. They also include dividing unit fractions by whole numbers and whole numbers by unit fractions, and then, finally, fractions by fractions. This careful and rigorous development is seldom seen in standards.

The standards develop place value quite well. Decimals are defined as special fractions and connected to place value. The goal of the operations is fluency with the standard algorithms for decimals.

Word problems are introduced early and appear throughout the standards, including multi-step problems. In the middle grades, the exemplary work with fractions and decimals is well utilized in the coverage of proportions, percents, rates, and ratios, which are covered with rigor and include many strong standards.

Area is begun nicely with:

> A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area (grade 3)
>
> Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths (grade 3)
>
> Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor (grade 4)

The high school material, despite its sometimes incoherent presentation, is often strong. The coverage of linear equations, which begins in eighth grade, includes some rigorous standards. For instance, the Common Core standards expect students to know that slope is well-defined, a rarity among standards:

> Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b (grade 8)

Quadratic functions are well covered. For geometry, while there are some issues (discussed below), much of the content is well covered. Classical theorems of geometry are explicitly included *and* proven:

> Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point (high school)

The important skills of arithmetic operations with rational expressions are included among the high school algebra standards:

> Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions (high school)

In addition, the standards cover most STEM-ready topics, including trigonometric identities, series, exponential functions, and inverse trigonometric functions.

Content Weaknesses

The foundation of K-12 mathematics is whole-number arithmetic. The basic number facts are the building blocks for such arithmetic, and instant recall of these facts should be required. Students should not need to concern themselves with computing such facts as they attempt to master more difficult techniques. The Common Core standards require memorization for the addition and multiplication facts but there is no mention of the corresponding subtraction and division facts.

Despite the good beginning for area, no formulas are developed for triangles and parallelograms.

Linear equations are missing point-slope form and an explicit mention of being able to find the equation of a line from two points.

Polar coordinates are not in the standards except briefly in a subservient role for complex numbers under "number and quantity":

> Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number (high school)

High school geometry has very good coverage of content, and proofs are included throughout the standards. There is, however, no obvious foundation for geometry, in part because axioms and postulates are never mentioned. Instead, the standards approach geometry through transformations. Unfortunately, it takes a good deal of work in Euclidean geometry (based on axioms) to work with transformations.

Content and Rigor Conclusion

The Common Core standards cover nearly all the essential content with appropriate rigor. In the elementary grades, arithmetic is well prioritized and generally well developed. In high school, there are a few issues with both content and organization, but most of the essential content is covered including the STEM-ready material. The standards receive a Content and Rigor score of seven points out of seven. (See *Common Grading Metric*, Appendix A.)

The Bottom Line

Despite their imperfections, the Common Core mathematics standards are far superior to those now in place in many states, districts, and classrooms. They are ambitious and challenging for students and educators alike. Accompanied by a properly aligned, content-rich curriculum, they provide K-12 teachers with a sturdy instructional framework for this most fundamental of subjects.