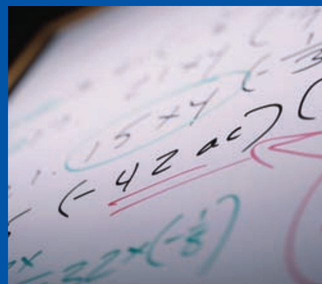

2005

The State of State MATH Standards

by **David Klein**

with **Bastiaan J. Braams, Thomas Parker, William Quirk, Wilfried Schmid,** *and* **W. Stephen Wilson**

Technical assistance from **Ralph A. Raimi** *and* **Lawrence Braden**

Analysis by **Justin Torres**

Foreword by **Chester E. Finn, Jr.**

THOMAS B. FORDHAM FOUNDATION

OUTSIDE THE BOX

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JANUARY 2005

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CONTENTS

Foreword

Chester E. Finn, Jr.

Two decades after the United States was diagnosed as "a nation at risk," academic standards for our primary and secondary schools are more important than ever—and their quality matters enormously.

In 1983, as nearly every American knows, the National Commission on Excellence in Education declared that "The educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people." Test scores were falling, schools were asking less of students, international rankings were slipping, and colleges and employers were complaining that many high school graduates were semi-literate. America was gripped by an education crisis that centered on weak academic achievement in its K-12 schools. Though that weakness had myriad causes, policy makers, business leaders, and astute educators quickly deduced that the surest cure would begin by spelling out the skills and knowledge that children *ought* to learn in school, i.e., setting standards against which progress could be tracked, performance be judged, and curricula (and textbooks, teacher training, etc.) be aligned. Indeed, the vast education renewal movement that gathered speed in the mid-1980s soon came to be known as "standards-based reform."

By 1989, President George H.W. Bush and the governors agreed on ambitious new national academic goals, including the demand that "American students will leave grades 4, 8, and 12 having demonstrated competency in challenging subject matter" in the core subjects of English, mathematics, science, history, and geography.

In response, states began to enumerate academic standards for their schools and students. In 1994, Washington added oomph to this movement (and more subjects to the "core" list) via the "Goals 2000" act and a revision of the federal Title I program.

Two years later, the governors and business leaders convened an education summit to map out a plan to strengthen K-12 academic achievement. The summiteers called for "new world-class standards" for U.S. schools. And by 1998, 47 states had outlined K-12 standards in mathematics.

But were they any good? We at the Thomas B. Fordham Foundation took it upon ourselves to find out. In early 1998, we published *State Math Standards*, written by the distinguished mathematician Ralph Raimi and veteran math teacher Lawrence Braden. Two years later, with many states having augmented or revised their academic standards, we published *The State of State Standards 2000*, whose math review was again conducted by Messrs. Raimi and Braden. It appraised the math standards of 49 states, conferring upon them an average grade of "C."

Raising the Stakes

Since that review, standards-based reform received a major boost from the No Child Left Behind act (NCLB) of 2002. Previously, Washington had encouraged states to set standards. Now, as a condition of federal education assistance, they *must* set them in math and reading (and, soon, science) in grades 3 through 8; develop a testing system to track student and school performance; and hold schools and school systems to account for progress toward universal proficiency as gauged by those standards.

Due mostly to the force of NCLB, more than 40 states have replaced, substantially revised, or augmented their K-12 math standards since our 2000 review. NCLB also raised the stakes attached to those standards. States, districts, and schools are now judged by how well they are educating their students and whether they are raising academic achievement for *all* students. The goal, now, is 100 percent proficiency. Moreover, billions of dollars in federal aid now hinge on whether states conscientiously hold their schools and districts to account for student learning.

Thus, a state's academic standards bear far more weight than ever before. These documents now provide the foundation for a complex, high-visibility, high-risk accountability system. "Standards-based" reform is the most powerful engine for education improvement in America, and all parts of that undertaking—including teacher preparation, textbook selection, and much more—are supposed to be aligned with a state's standards. If that foundation is sturdy, such reforms may succeed; if it's weak, uneven, or cracked, reforms erected atop it will be shaky and, in the end, could prove worse than none at all.

Constancy and Change

Mindful of this enormous burden on state standards, and aware that most of them had changed substantially since our last review, in 2004 we initiated fresh appraisals in mathematics and English, the two subjects at NCLB's heart. To lead the math review, we turned to Dr. David Klein, a professor of mathematics at California State University, Northridge, who has long experience in K-12 math issues. We encouraged him to recruit an expert panel of fellow mathematicians to collaborate in this ambitious venture, both to expose states' standards to more eyes, thus improving the reliability and consistency of the ratings, and to share the work burden.

Dr. Klein outdid himself in assembling such a panel of five eminent mathematicians, identified on page 127. We could not be more pleased with the precision and rigor that they brought to this project.

It is inevitable, however, that when reviewers change, reviews will, too. Reviewing entails judgment, which is inevitably the result of one's values and priorities as well as expert knowledge and experience.

In all respects but one, though, Klein and his colleagues strove to replicate the protocols and criteria developed by Raimi and Braden in the two earlier Fordham studies. Indeed, they asked Messrs. Raimi and Braden to advise this project and provide insight into the challenges the reviewers faced in this round. Where they intentionally deviated from the 1998 and 2000 reviews—and did so with the encouragement and assent of Raimi and Braden—was in weighting the four major criteria against which state standards are evaluated.

As Klein explains on page 9, the review team concluded that today the single most important consideration for statewide math standards is the selection (and accuracy) of their content coverage. Accordingly, content now counts for two-fifths of a state's grade, up from 25 percent in earlier evaluations. The other three criteria (clarity, mathematical reasoning, and the absence of "negative qualities") count for 20 percent each. If the content isn't there (or is wrong), our review team judged, such factors as clarity of expression cannot compensate. Such standards resemble clearly written recipes that use the wrong ingredients or combine them in the wrong proportions.

Glum Results

Though the rationale for changing the emphasis was not to punish states, only to hold their standards to higher expectations at a time when NCLB is itself raising the bar throughout K-12 education, the shift in criteria contributed to an overall lowering of state "grades." Indeed, as the reader will see in the following pages, the essential finding of this study is that the overwhelming majority of states today have sorely inadequate math standards. Their average grade is a "high D"—and just six earn "honors" grades of A or B, three of each. Fifteen states receive Cs, 18 receive Ds and 11 receive Fs. (The District of Columbia is included in this review but Iowa is not because it has no statewide academic standards.)

Tucked away in these bleak findings is a ray of hope. Three states—California, Indiana, and Massachusetts—have first-rate math standards, worthy of emulation. If they successfully align their other key policies (e.g., assessments, accountability, teacher preparation, textbooks, graduation requirements) with those fine standards, and if their schools and teachers succeed in instructing pupils in the skills and content specified in those standards, they can look forward to a top-notch K-12 math program and likely success in achieving the lofty goals of NCLB.

Yes, it's true. Central as standards are, getting them right is just the first element of a multi-part education reform strategy. Sound statewide academic standards are necessary but insufficient for the task at hand.

In this report, we evaluate that necessary element. Besides applying the criteria and rendering judgments on the standards, Klein and his team identified a set of widespread failings that weaken the math standards of many states. (These are described beginning on page 9 and crop up repeatedly in the state-specific report cards that begin on page 37.) They also trace the source of much of this weakness to states' unfortunate embrace of the advice of the National Council of Teachers of Mathematics (NCTM), particularly the guidance supplied in that organization's wrongheaded 1989 standards. (A later NCTM publication made partial amends, but these came too late for the standards—and the children—of many states.)

Setting It Right

Klein also offers four recommendations to state policy makers and others wishing to strengthen their math standards. Most obviously, states should cease and desist from doing the misguided things that got them in trouble in the first place (such as excessive emphasis on calculators and manipulatives, too little attention to fractions and basic arithmetic algorithms). They suggest that states not be afraid to follow the lead of the District of Columbia, whose new superintendent announced in mid-autumn 2004 that he would simply jettison D.C.'s woeful standards and adopt the excellent schema already in use in Massachusetts. That some states already have fine standards proves that states can develop them if they try. But if, as I think, there's no meaningful difference between good math education in North Carolina and Oregon or between Vermont and Colorado, why shouldn't states avoid a lot of heavy lifting, swallow a wee bit of pride, and duplicate the standards of places that have already got it right?

Klein and his colleagues insist that states take arithmetic instruction seriously in the elementary grades and ensure that it is mastered before a student proceeds into high school. As Justin Torres remarks in his *Memo to Policy Makers*, "It says something deeply unsettling about the parlous state of math education in these United States that the arithmetic point must even be raised—but it must." The recent results of two more international studies (PISA and TIMSS) make painfully clear once again that a vast swath of U.S. students cannot perform even simple arithmetic calculations. This ignorance has disastrous implications for any effort to train American students in the higher-level math skills needed to succeed in today's jobs. No wonder we're now outsourcing many of those jobs to lands with greater math prowess—or importing foreign students to fill them on U.S. shores.

Klein makes one final recommendation that shouldn't need to be voiced but does: Make sure that future math standards are developed by people who know lots and lots of math, including a proper leavening of true mathematicians. One might suppose states would figure this out for themselves, but it seems that many instead turned over the writing of their math standards to people with a shaky grip of the discipline itself.

One hopes that state leaders will heed this advice. One hopes, especially, that many more states will fix their math standards before placing upon them the added weight of new high school reforms tightly joined to statewide academic standards, as President Bush is urging. Even now, one wonders whether the praiseworthy goals of NCLB can be attained if they're aligned with today's woeful math standards—and whether the frailties that were exposed yet again by 2004's international studies can be rectified unless the standards that drive our K-12 instructional system become world-class.

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Many people deserve thanks for their roles in the creation of this report. David Klein did an awesome amount of high-quality work—organizational, intellectual, substantive, and editorial. Our hat is off to him, the more so for having persevered despite a painful personal loss this past year. We are grateful as well to Bastiaan J. Braams, Thomas Parker, William Quirk, Wilfried Schmid, and W. Stephen Wilson, Klein's colleagues in this review, as well as to Ralph Raimi and Lawrence Braden for excellent counsel born of long experience.

At the Fordham end, interns Carolyn Conner and Jess Castle supplied valuable research assistance and undertook the arduous task of gathering 50 sets of standards from websites and state departments of education. Emilia Ryan expertly designed this volume. And research director Justin Torres oversaw the whole venture from initial conceptualization through execution, revision, and editing, combining a practiced editor's touch with an analyst's rigor, a diplomat's people skills, and a manager's power of organization. Most of the time he even clung to his sense of humor!

. . . .

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Chester E. Finn, Jr.
President
Washington, D.C.
January 2005

Executive Summary

Statewide academic standards not only provide goal posts for teaching and learning across all of a state's public schools; they also drive myriad other education policies. Standards determine—or should determine—the content and emphasis of tests used to track pupil achievement and school performance; they influence the writing, publication, and selection of textbooks; and they form the core of teacher education programs. The quality of a state's K-12 academic standards thus holds far-reaching consequences for the education of its citizens, the more so because of the federal No Child Left Behind act. That entire accountability edifice rests upon them—and the prospect of extending its regimen to include high schools further raises the stakes.

This is the third review of state math standards by the Thomas B. Fordham Foundation. (Earlier studies were released in 1998 and 2000.) Here, states are judged by the same criteria: the standards' clarity, content, and sound mathematical reasoning, and the absence of negative features. This report differs, however, in its weighting of those criteria. Content now accounts for 40 percent of a state's total score, compared to 25 percent in prior reports. The consensus of the evaluating panel of mathematicians is that this revised weighting properly reflects what matters most in K-12 standards today.

Major Findings

With greater weight attached to mathematical content, it is not surprising that the grades reported here are lower than in 2000. We were able to confer A grades on just three states: California, Indiana, and Massachusetts. Alabama, New Mexico, and Georgia—all receiving Bs—round out the slim list of "honors" states. The national average grade is D, with 29 states receiving Ds or Fs and 15 getting Cs.

Common Problems

Why do so many state mathematics standards come up short? Nine major problems are widespread.

Fig. 1: 2005 Results, alphabetized

	Score	Grade
Alabama	2.97	B
Alaska	1.32	D
Arizona	2.00	C
Arkansas	0.72	F
California	3.89	A
Colorado	1.37	D
Connecticut	0.47	F
Delaware	0.54	F
DC	1.37	D
Florida	0.93	F
Georgia	2.53	B
Hawaii	0.43	F
Idaho	1.10	D
Illinois	1.80	C
Indiana	3.82	A
Iowa	-	-
Kansas	0.83	F
Kentucky	1.80	C
Louisiana	1.78	C
Maine	1.35	D
Maryland	1.77	C
Massachusetts	3.30	A
Michigan	2.00	C
Minnesota	1.67	D
Mississippi	1.67	D
Missouri	0.57	F
Montana	1.00	D
Nebraska	1.42	D
Nevada	1.77	C
New Hampshire	0.70	F
New Jersey	1.15	D
New Mexico	2.67	B
New York	2.08	C
North Carolina	1.82	C
North Dakota	1.80	C
Ohio	1.43	D
Oklahoma	1.97	C
Oregon	1.35	D
Pennsylvania	1.28	D
Rhode Island	0.67	F
South Carolina	1.32	D
South Dakota	1.80	C
Tennessee	1.70	D
Texas	1.80	C
Utah	1.13	D
Vermont	1.20	D
Virginia	1.97	C
Washington	0.57	F
West Virginia	2.35	C
Wisconsin	1.50	D
Wyoming	0.98	F
National Average	1.59	D

Fig. 2: 2005 Results, ranked		
	Score	Grade
California	3.89	A
Indiana	3.82	A
Massachusetts	3.30	A
Alabama	2.97	B
New Mexico	2.67	B
Georgia	2.53	B
West Virginia	2.35	C
New York	2.08	C
Michigan	2.00	C
Arizona	2.00	C
Oklahoma	1.97	C
Virginia	1.97	C
North Carolina	1.82	C
South Dakota	1.80	C
Texas	1.80	C
Illinois	1.80	C
Kentucky	1.80	C
North Dakota	1.80	C
Louisiana	1.78	C
Maryland	1.77	C
Nevada	1.77	C
Tennessee	1.70	D
Minnesota	1.67	D
Mississippi	1.67	D
National Average	1.59	D
Wisconsin	1.50	D
Ohio	1.43	D
Nebraska	1.42	D
Colorado	1.37	D
DC	1.37	D
Maine	1.35	D
Oregon	1.35	D
Alaska	1.32	D
South Carolina	1.32	D
Pennsylvania	1.28	D
Vermont	1.20	D
New Jersey	1.15	D
Utah	1.13	D
Idaho	1.10	D
Montana	1.00	D
Wyoming	0.98	F
Florida	0.93	F
Kansas	0.83	F
Arkansas	0.72	F
New Hampshire	0.70	F
Rhode Island	0.67	F
Missouri	0.57	F
Washington	0.57	F
Delaware	0.54	F
Connecticut	0.47	F
Hawaii	0.43	F
Iowa	-	-

1. Calculators

One of the most debilitating trends in current state math standards is their excessive emphasis on calculators. Most standards documents call upon students to use them starting in the elementary grades, often beginning with Kindergarten. Calculators enable students to do arithmetic quickly, without thinking about the numbers involved in a calculation. For this reason, using them in a high school science class, for example, is perfectly sensible. But for elementary students, the main goal of math education is to get them to think about numbers and to learn arithmetic. Calculators defeat that purpose. With proper restriction and guidance, calculators can play a positive role in school mathematics, but such direction is almost always missing in state standards documents.

2. Memorization of Basic Number Facts

Memorizing the "basic number facts," i.e., the sums and products of single-digit numbers and the equivalent subtraction and division facts, frees up working memory to master the arithmetic algorithms and tackle math applications. Students who do not memorize the basic number facts will founder as more complex operations are required, and their progress will likely grind to a halt by the end of elementary school. There is no *real* mathematical fluency without memorization of the most basic facts. The many states that do not require such memorization of their students do them a disservice.

3. The Standard Algorithms

Only a minority of states explicitly require knowledge of the standard algorithms of arithmetic for addition, subtraction, multiplication, and division. Many states identify no methods for arithmetic, or, worse, ask students to invent their own algorithms or rely on *ad hoc* methods. The standard algorithms are powerful theorems and they are *standard* for a good reason: They are guaranteed to work for all problems of the type for which they were designed. Knowing the standard algorithms, in the sense of being able to use them and understanding how and why they work, is the most sophisticated mathematics that an elementary school student is likely to grasp, and it is a foundational skill.

4. Fraction Development

In general, too little attention is paid to the coherent development of fractions in the late elementary and early middle grades, and there is not enough emphasis on paper-and-pencil calculations. A related topic at the high school level that deserves much more attention is the arithmetic of rational functions. This is crucial for students planning university studies in math, science, or engineering-related majors. Many state standards would also benefit from greater emphasis on completing the square of quadratic polynomials, including a derivation of the quadratic formula, and applications to graphs of conic sections.

5. Patterns

The attention given to patterns in state standards verges on the obsessive. In a typical document, students are asked, across many grade levels, to create, identify, examine, describe, extend, and find "the rule" for repeating, growing, and shrinking patterns, where the patterns may be found in numbers, shapes, tables, and graphs. We are not arguing for elimination of all standards calling upon students to recognize patterns. But the attention given to patterns is far out of balance with the actual importance of patterns in K-12 mathematics.

6. Manipulatives

Manipulatives are physical objects intended to serve as teaching aids. They can be helpful in introducing new concepts for elementary pupils, but too much use of them runs the risk that students will focus on the manipulatives more than the math, and even come to depend on them. In the higher grades, manipulatives can undermine important educational goals. Yet many state standards recommend and even require the use of a dizzying array of manipulatives in counterproductive ways.

7. Estimation

Fostering estimation skills in students is a commendable goal shared by all state standards documents. However, there is a tendency to overemphasize estimation at the expense of exact arithmetic calculations. For simple subtraction, the correct answer is the only reasonable answer. The notion of "reasonableness" might be addressed in the first and second grades in connection with measurement, but not in connection with arithmetic of small whole numbers. Care should be taken not to substitute estimation for exact calculations.

8. Probability and Statistics

With few exceptions, state standards at all grade levels include strands devoted to probability and statistics. Such standards almost invariably begin by Kindergarten. Yet sound math standards delay the introduction of probability until middle school, then proceed quickly by building on students' knowledge of fractions and ratios. Many states also include data collection standards that are excessive. Statistics and probability requirements often crowd out important topics in algebra and geometry. Students would be better off learning, for example, rational function arithmetic, or how to complete the square for a quadratic polynomial—topics frequently missing or abridged.

9. Mathematical Reasoning and Problem-Solving

Problem-solving is an indispensable part of learning mathematics and, ideally, straightforward practice problems should gradually give way to more difficult problems as students master more skills. Children should solve single-step word problems in the earliest grades and deal with increasingly more challenging, multi-step problems as they progress. Unfortunately, few states offer standards that guide the development of problem-solving in a useful way. Likewise, mathematical reasoning should be an integral part of the content at all grade levels. Too many states fail to develop important prerequisites before introducing advanced topics such as calculus. This degrades mathematics standards into what might be termed "math appreciation."

How Can States Improve Their Standards?

We offer four suggestions to states wishing to strengthen their K-12 math standards:

Replace the authors of weak standards documents with people who thoroughly understand mathematics, including university professors from math departments. Many states have delegated standards development to "math educators" or "curriculum experts" with inadequate backgrounds in the discipline. States must make actual mathematics competency a prerequisite for inclusion on the panels that draft standards.

Develop coherent arithmetic standards that emphasize both conceptual understanding *and* computational fluency. Most states have failed to develop acceptable standards even for arithmetic, the most elementary but also most important branch of mathematics. It is impossible to develop a coherent course of study in K-12 mathematics without a solid foundation of arithmetic.

Avoid, or rectify, "common problems." We have identified nine shortcomings that recur in many state standards, such as overuse of calculators and manipulatives, overemphasis on patterns and statistics, etc. Obviously, standards documents would be improved if states avoided those problems.

Consider borrowing a complete set of high-quality math standards from a top-scoring state. There is no need to reinvent this wheel. California, Indiana, and Massachusetts have done this expertly. Other states could benefit from their success.

The State of State Math Standards 2005

David Klein

Statewide academic standards are important, not only because they provide goal posts for teaching and learning, but also because they drive education policies. Standards determine—or should determine—the content and emphasis of tests used to measure student achievement; they influence the selection of textbooks; and they form the core of teacher education programs. The quality of a state's K-12 academic standards has far-reaching consequences for the education of its citizens.

The quality of state mathematics standards was the subject of two previous reports from the Thomas B. Fordham Foundation, both authored by Ralph Raimi and Lawrence Braden. The first, published in March 1998 (which we refer to as Fordham I), was a pioneering work. Departing from previous such undertakings, it exposed the shocking inability of most state education bureaucracies even to describe what public schools should teach students in math classes. The average national grade was a D. Only three states received A grades, and more than half received grades of D or F. "On the whole," wrote the authors in 1998, "the nation flunks."

The Fordham I grades were based on numerical scores in four categories: clarity, content, reasoning, and negative qualities. Using these same criteria, the Foundation released Raimi and Braden's second report in January 2000 (which we refer to as Fordham II). It evaluated 34 new or revised state documents and retained the original evaluations of 15 states whose math standards had not changed since Fordham I. The result was a national average grade of C, an apparent improvement. However, Fordham II, like Fordham I, cautioned readers not to take the overall average grade as a definitive description of performance, and to read the scores (0 to 4 possible points) for the four criteria separately, to arrive at an understanding of the result. Ralph Raimi made clear in his introduction to Fordham II that much of the increase of the final grades was due to improved clarity. States had improved upon prose that Raimi termed "appallingly vague, so general as to be unusable for guiding statewide testing or the choice of textbooks." The result was that many states had by the time of Fordham II achieved higher overall grades through little more than a clearer exposition of standards with defective mathematical content.

Major Findings

The criteria for evaluation used in this report are the same as in Fordham I and II. For the reader's convenience, these criteria are defined and described in the next section. *However, this report differs from Fordham I and II in that the relative weights of the criteria have been changed*. At the suggestion of Raimi and Braden, we increased the weight of the content criterion and reduced uniformly the weights of the other three criteria: clarity, reason, and negative qualities. Content now accounts for 40 percent of a state's total score, compared to 25 percent in Fordham I and II. This affects the final numerical scores upon which our grades are based and, in some cases, results in lower grades, especially for states that benefited from higher "clarity" scores in Fordham II. The individual state reports beginning on page 37 include numerical scores for each criterion. The Appendix, on page 123, also includes numerical scores for subcategories of these four criteria.

The consensus of the evaluating panel of mathematicians is that this weighting properly reflects what is most important in K-12 standards in 2005. Content is what matters most in state standards; clear but insubstantial expectations are insufficient.

With the greater weight attached to mathematical content in this report, it is not surprising that our grades are lower than those of Fordham II. In fact, our grade distribution more closely resembles that of Fordham I. We assigned A, or "excellent," grades to only three states: California, Indiana, and Massachusetts. The national average grade is D, or "poor," with most states receiving D or F grades. The table below shows the scores and grade assignments for 49 states and the District of

Columbia (which for purposes of this report we refer to as a state). Only Iowa is missing, because it has no standards documents.

Besides the different weighting of criteria for evaluation, another caveat for those wanting to compare Fordham I and II with this report to identify trends over time is the change of authorship. None of the mathematicians who scored and evaluated the state math standards in 2005 had any involvement in Fordham I and II. However, Ralph Raimi and Lawrence Braden served as advisers for this project, and helped to resolve many technical questions that arose in the course of evaluating state documents. We describe this interaction in greater detail in the section, "Methods and Procedures," on page 121.

Common Problems

What are some of the reasons that so many state mathematics standards come up short? We discuss here nine problems found in many, and in some cases most, of the standards documents that we reviewed.

Calculators

One of the most debilitating trends in current state math standards is overemphasis of calculators. The majority of state standards documents call upon students to use calculators starting in the elementary grades, often beginning in Kindergarten and sometimes even in pre-Kindergarten. For example, the District of Columbia requires that the pre-Kindergarten student "demonstrates familiarity with basic calculator keys." New Hampshire directs Kindergarten teachers to "allow students to explore one-more-than and one-less-than patterns with a calculator" and first grade teachers "have students use calculators to explore the operation of addition and subtraction," along with much else. In Georgia, first-graders "determine the most efficient way to solve a problem (mentally, paper/pencil, or calculator)." According to New Jersey's policy:

Calculators can and should be used at all grade levels to enhance student understanding of mathematical concepts. The majority of questions on New Jersey's

Fig. 3: State Grades, Alphabetical Order

STATE	Clarity	Content	Reason	Negative Qualities	Final G.P.A.	2005 GRADE
AL	3.00	3.17	2.00	3.50	2.97	B
AK	2.00	1.17	0.50	1.75	1.32	D
AZ	2.00	2.00	2.00	2.00	2.00	C
AR	1.50	0.67	0.00	0.75	0.72	F
CA	3.83	3.94	3.83	3.92	3.89	A
CO	1.00	1.67	1.00	1.50	1.37	D
CT	0.67	0.33	0.00	1.00	0.47	F
DE	0.83	0.67	0.50	0.00	0.54	F
DC	1.67	1.33	1.50	1.00	1.37	D
FL	1.33	0.67	1.50	0.50	0.93	F
GA	3.33	2.67	2.00	2.00	2.53	B
HI	1.00	0.33	0.00	0.50	0.43	F
ID	1.67	0.67	1.00	1.50	1.10	D
IL	1.50	2.00	1.00	2.50	1.80	C
IN	3.67	3.83	4.00	3.75	3.82	A
IA	-	-	-	-	-	-
KS	1.67	0.94	0.33	0.25	0.83	F
KY	1.83	2.33	1.00	1.50	1.80	C
LA	2.00	2.33	1.00	1.25	1.78	C
ME	1.17	1.17	0.50	2.75	1.35	D
MD	2.00	1.67	1.50	2.00	1.77	C
MA	3.67	3.67	2.00	3.50	3.30	A
MI	2.17	1.67	2.00	2.50	2.00	C
MN	2.00	1.67	1.00	2.00	1.67	D
MS	1.33	2.00	1.00	2.00	1.67	D
MO	0.67	0.33	1.00	0.50	0.57	F
MT	1.00	1.00	0.00	2.00	1.00	D
NE	1.72	1.28	0.67	2.17	1.42	D
NV	2.17	1.33	1.50	2.50	1.77	C
NH	1.17	0.67	0.00	1.00	0.70	F
NJ	2.17	1.17	0.50	0.75	1.15	D
NM	3.00	2.67	2.00	3.00	2.67	B
NY	1.50	2.33	2.00	2.25	2.08	C
NC	2.33	1.50	1.50	2.25	1.82	C
ND	2.33	1.33	1.00	3.00	1.80	C
OH	2.00	1.33	1.00	1.50	1.43	D
OK	2.17	1.83	1.50	2.50	1.97	C
OR	2.50	1.00	0.00	2.25	1.35	D
PA	1.33	1.17	1.00	1.75	1.28	D
RI	1.00	0.67	0.00	1.00	0.67	F
SC	1.00	1.67	1.50	0.75	1.32	D
SD	2.17	1.67	1.00	2.50	1.80	C
TN	1.83	1.33	2.00	2.00	1.70	D
TX	2.67	1.67	1.00	2.00	1.80	C
UT	1.83	1.17	0.50	1.00	1.13	D
VT	1.33	1.00	0.67	2.00	1.20	D
VA	2.83	2.00	1.50	1.50	1.97	C
WA	0.33	1.00	0.50	0.00	0.57	F
WV	2.00	2.50	3.00	1.75	2.35	C
WI	1.67	1.67	1.00	1.50	1.50	D
WY	1.00	0.83	0.00	2.25	0.98	F
Average	1.85	1.57	1.15	1.79	1.59	D

(A = 4.00 - 3.25; B = 3.24 - 2.50; C = 2.49 - 1.75; D = 1.74 - 1.00; F = 0.99 - 0.00)

new third- and fourth-grade assessments in mathematics will assume student access to at least a four-function calculator.

Alaska's standards explicitly call upon third-graders to determine answers "to real-life situations, paper/pencil computations, or calculator results by finding 'how many' or 'how much' to 50." For references and a nearly endless supply of examples, we refer the reader to the state reports that follow.

Calculators enable students to do arithmetic quickly, without thinking about the numbers involved in a calculation. For this reason, using calculators in a high school science class, for example, is perfectly sensible. There, the speed and efficiency of a calculator keep the focus where it belongs, on science, much as the slide rule did in an earlier era. At that level, laborious hand calculations have no educational value, because high school science students already know arithmetic—or they should.

By contrast, elementary school students are still learning arithmetic. The main goal of elementary school mathematics education is to get students to think about numbers and to learn arithmetic. Calculators defeat that purpose. They allow students to arrive at answers without thinking. Hand calculations and mental mathematics, on the other hand, force students to develop an intuitive understanding of place value in the decimal system, and of fractions. Consider the awkwardly written Alaska standard cited above. Allowing third-graders to use calculators to find sums to 50 is not only devoid of educational value, it is a barrier to sound mathematics education. Some state standards even call for the use of fraction calculators in elementary or middle school, potentially compromising facility in rational number arithmetic, an essential prerequisite for high school algebra.

An implicit assumption of most state standards is that students need practice using calculators over a period of years, starting at an early age. Thus, very young children are exposed to these machines in order to achieve familiarity and eventual competence in their use. But anyone can rapidly learn to press the necessary buttons on a calculator. Standards addressing "calculator skills" have no more place in elementary grade standards than do standards addressing skills for dialing telephone numbers.

With proper restriction and guidance, calculators can play a positive role in school mathematics, but such direction is almost always missing in state standards documents. A rare exception is the California *Framework*, which warns against over-use, but also identifies specific topics, such as compound interest, for which the calculator is appropriate. As in many European and Asian countries, the California curriculum does not include calculators for any purpose until the sixth grade, and thereafter only with prudence.

Many states diminish the quality of their standards by overemphasis of calculators and other technology, not only in the lower grades, but even at the high school level. Standards calling for students to use graphing calculators to plot straight lines are not uncommon. Students should become skilled in graphing linear functions by hand, and be cognizant of the fact that only two points are needed to determine the entire graph of a line. This fundamental fact is easily camouflaged by the obsessive use of graphing technology. Similarly, the use of graphing calculators to plot conic sections can easily and destructively supplant a mathematical idea of central importance for this topic and others: completing the square.

Memorization of the Basic Number Facts

We use the term "basic number facts" to refer to the sums and products of single-digit numbers and to the equivalent subtraction and division facts. Students need to memorize the basic number facts because doing so frees up working memory required to master the arithmetic algorithms and tackle applications of mathematics. Research in cognitive psychology points to the value of automatic recall of the basic facts.[1] Students who do not memorize the basic number facts will founder as more complex operations are required of them, and their progress in mathematics will likely grind to a halt by the end of elementary school.

[1] A cogent summary of some of that research appears on pages 150-151 and 224 of *The Schools We Need: And Why We Don't Have Them*, by E.D. Hirsch, Jr., Doubleday, 1996.

Unfortunately, many states do not explicitly require students to memorize the basic number facts. For example, rather than memorizing the addition and subtraction facts, Utah's second-graders "compute accurately with basic number combinations for addition and subtraction facts to eighteen," and, rather than memorize the multiplication and division facts, Oregon's fourth-graders are only required to "apply with fluency efficient strategies for determining multiplication and division facts 0-9." Computing accurately that $6 + 7 = 13$ and using efficient strategies to calculate that $6 x 7 = 42$ is not the same as memorizing these facts. We are not suggesting that the meaning of the facts should not also be taught. Students should of course understand the meaning of the four arithmetic operations, as well as ways in which the basic number facts can be recovered without memory. All are important. But there is no *real* fluency without memorization of the most basic facts. The states that decline to require this do their students a disservice.

The Standard Algorithms

Only a minority of states explicitly require knowledge of the standard algorithms of arithmetic for addition, subtraction, multiplication, and division. Instead, many states do not identify any methods for arithmetic, or worse, ask students to invent their own algorithms or rely on *ad hoc* methods. One of Connecticut's standards documents advises,

> *Instructional activities and opportunities need to focus on developing an understanding of mathematics as opposed to the memorization of rules and mechanical application of algorithms.*

This is insufficient. Specialized methods for mental math work well in some cases but not in others, and it is unwise for schools to leave students with untested, private algorithms for arithmetic operations. Such procedures might be valid only for a subclass of problems. The standard algorithms are powerful theorems and they are *standard* for a good reason: they are guaranteed to work for all problems of the type for which they were designed.

Knowing the standard algorithms, in the sense of being able to use them and understanding how and why they work, is the most sophisticated mathematics that an elementary school student is likely to grasp. Students who have mastered these algorithms gain confidence in their ability to compute. They know that they can solve any addition, subtraction, multiplication, or division problem without relying on a mysterious black box, such as a calculator. Moreover, the ability to execute the arithmetic operations in a routine manner helps students to think more conceptually. As their use of the standard algorithms becomes increasingly automatic, students come to view expressions such as $6485 - 3689$ as a single number that can be found easily, rather than thinking of it as a complicated problem in itself. If mathematical thinking is the goal, the standard algorithms are a valuable part of the curriculum.

A wide variety of algorithms are used in mathematics and engineering, and our technological age surrounds us with machines that depend on the algorithms programmed into them. Students who are adept with the most important and fundamental examples of algorithms—the standard algorithms of arithmetic—are well positioned to understand the meaning and uses of other algorithms in later years.

One benefit of learning the long division algorithm is that it requires estimation of quotients at each stage. If the next digit placed in the (trial) answer is too large or too small, that stage has to be done over again, and the error is made visible by the procedure. Number sense and estimation skills are reinforced in this way. The long division algorithm illustrates an important idea in mathematics: repeated estimations leading to increasingly accurate approximations.

The long division algorithm has applications that go far beyond elementary school arithmetic. At the middle school level, it can be used to explain why rational numbers have repeating decimals. This leads to an understanding of irrational, and therefore real numbers. Division is also central to the Euclidean Algorithm for the calculation of the greatest common divisor of two integers. In high school algebra, the long division algorithm, in slightly modified form, is used for division of polynomials. At the university level, the algorithm is generalized to accommodate division of power series and it is also important in advanced abstract algebra. Experience with the long division algorithm in elementary school thus lays the groundwork for advanced topics in mathematics.

Overemphasized and Underemphasized Topics

There is remarkable consistency among the states in topics that are overemphasized and underemphasized.

In general, we found too little attention paid to the coherent development of fractions in the late elementary and early middle school grades, and not enough emphasis on paper-and-pencil calculations. A related topic at the high school level that deserves much more emphasis is the arithmetic of rational functions. This is crucial for students planning university studies in math-related majors, including engineering and the physical and biological sciences. They will need facility in addition, subtraction, multiplication, and division of rational functions, including long division of polynomials. The most important prerequisite for this frequently missing topic in state standards is the arithmetic of fractions. Many state standards would also benefit from greater emphasis on completing the square of quadratic polynomials, including a derivation of the quadratic formula, and applications to graphs of conic sections.

Among topics that receive too much emphasis in state standards are patterns, use of manipulatives, estimation, and probability and statistics. We discuss each of these in turn.

Patterns

The attention given to patterns in state standards verges on the obsessive. In a typical state document, students are asked, through a broad span of grade levels, to create, identify, examine, describe, extend, and find "the rule" for repeating, growing, and shrinking patterns, as well as where the patterns may be found in numbers, shapes, tables, and graphs. Thus, first-graders in Maryland are required to "recognize the difference between patterns and non-patterns." How this is to be done, and what

Fig. 4: State Grades in Descending Order

STATE	Clarity	Content	Reason	Negative Qualities	Final G.P.A.	2005 GRADE
CA	3.83	3.94	3.83	3.92	3.89	A
IN	3.67	3.83	4.00	3.75	3.82	A
MA	3.67	3.67	2.00	3.50	3.30	A
AL	3.00	3.17	2.00	3.50	2.97	B
NM	3.00	2.67	2.00	3.00	2.67	B
GA	3.33	2.67	2.00	2.00	2.53	B
WV	2.00	2.50	3.00	1.75	2.35	C
NY	1.50	2.33	2.00	2.25	2.08	C
MI	2.17	1.67	2.00	2.50	2.00	C
AZ	2.00	2.00	2.00	2.00	2.00	C
OK	2.17	1.83	1.50	2.50	1.97	C
VA	2.83	2.00	1.50	1.50	1.97	C
NC	2.33	1.50	1.50	2.25	1.82	C
SD	2.17	1.67	1.00	2.50	1.80	C
TX	2.67	1.67	1.00	2.00	1.80	C
IL	1.50	2.00	1.00	2.50	1.80	C
KY	1.83	2.33	1.00	1.50	1.80	C
ND	2.33	1.33	1.00	3.00	1.80	C
LA	2.00	2.33	1.00	1.25	1.78	C
MD	2.00	1.67	1.50	2.00	1.77	C
NV	2.17	1.33	1.50	2.50	1.77	C
TN	1.83	1.33	2.00	2.00	1.70	D
MN	2.00	1.67	1.00	2.00	1.67	D
MS	1.33	2.00	1.00	2.00	1.67	D
Average	1.85	1.57	1.15	1.79	1.59	D
WI	1.67	1.67	1.00	1.50	1.50	D
OH	2.00	1.33	1.00	1.50	1.43	D
NE	1.72	1.28	0.67	2.17	1.42	D
CO	1.00	1.67	1.00	1.50	1.37	D
DC	1.67	1.33	1.50	1.00	1.37	D
ME	1.17	1.17	0.50	2.75	1.35	D
OR	2.50	1.00	0.00	2.25	1.35	D
AK	2.00	1.17	0.50	1.75	1.32	D
SC	1.00	1.67	1.50	0.75	1.32	D
PA	1.33	1.17	1.00	1.75	1.28	D
VT	1.33	1.00	0.67	2.00	1.20	D
NJ	2.17	1.17	0.50	0.75	1.15	D
UT	1.83	1.17	0.50	1.00	1.13	D
ID	1.67	0.67	1.00	1.50	1.10	D
MT	1.00	1.00	0.00	2.00	1.00	D
WY	1.00	0.83	0.00	2.25	0.98	F
FL	1.33	0.67	1.50	0.50	0.93	F
KS	1.67	0.94	0.33	0.25	0.83	F
AR	1.50	0.67	0.00	0.75	0.72	F
NH	1.17	0.67	0.00	1.00	0.70	F
RI	1.00	0.67	0.00	1.00	0.67	F
MO	0.67	0.33	1.00	0.50	0.57	F
WA	0.33	1.00	0.50	0.00	0.57	F
DE	0.83	0.67	0.50	0.00	0.54	F
CT	0.67	0.33	0.00	1.00	0.47	F
HI	1.00	0.33	0.00	0.50	0.43	F
IA	-	-	-	-	-	-

(A = 4.00 - 3.25; B = 3.24 - 2.50; C = 2.49 - 1.75; D = 1.74 - 1.00; F = 0.99 - 0.00)

exactly is meant by a pattern, is anyone's guess. Florida's extensive requirements for the study of patterns call upon second-graders to use "a calculator to explore and solve number patterns"; identify "patterns in the real-world (for example, repeating, rotational, tessellating, and patchwork)"; and explain "generalizations of patterns and relationships," among other requirements.

The following South Dakota fourth-grade standard is an example of false doctrine (a notion explained in greater detail on page 34) that is representative of standards in many other state documents.

> *Students are able to solve problems involving pattern identification and completion of patterns. Example: What are the next two numbers in the sequence? Sequence: ...*

The sequence "1, 3, 7, 13, __, __" is then given. The presumption here is that there is a unique correct answer for the next two terms of the sequence, and by implication, for other number sequences, such as: 2, 4, 6, __, ___, and so forth. How should the blanks be filled for this example? The pattern might be continued in this way: 2, 4, 6, 8, 10, etc. But it might also be continued this way: 2, 4, 6, 2, 4, 6, 2, 4, 6. Other continuations include: 2, 4, 6, 4, 2, 4, 6, 4, 2, or 2, 4, 6, 5, 2, 4, 6, 5. Similarly, for the example in the South Dakota standard, the continuation might proceed as 1, 3, 7, 13, 21, 31, or as 1, 3, 7, 13, 1, 3, 7, 13, or in any other way. Given only the first four terms of a pattern, there are infinitely many systematic, and even polynomial, ways to continue the pattern, and there are *no possible incorrect* fifth and sixth terms. Advocating otherwise is both false and confusing to students. Such problems, especially when posed on examinations, misdirect students to conclude that mathematics is about mind reading: To get the correct answer, it is necessary to know what the teacher wants. Without a rule for a pattern, there is no mathematically correct or incorrect way to fill in the missing numbers.

Typical strands in state standards documents are "Patterns, Functions, and Algebra," "Patterns and Relationships," "Patterns, Relations, and Algebra," "Patterns and Relationships," and so forth. As these strand titles suggest, there is a tendency among the states to conflate the study of algebra with the exploration of patterns. For example, Wyoming's entire "Algebraic Concepts and Relationships" strand for fourth grade consists of three standards, all devoted to the study of patterns:

1. *Students recognize, describe, extend, create, and generalize patterns by using manipulatives, numbers, and graphic representations.*
2. *Students apply knowledge of appropriate grade level patterns when solving problems.*
3. *Students explain a rule given a pattern or sequence.*

An obscure Montana high school algebra standard requires students to "use algebra to represent patterns of change." South Carolina's seventh-graders are asked to:

> *Explain the use of a variable as a quantity that can change its value, as a quantity on which other values depend, and as generalization of patterns.*

The convoluted standard above illustrates several generic deficiencies of state algebra standards. The notion that algebra is the study of patterns is not only wrong, it shrouds the study of algebra in mystery and can lead to nonsensical claims like the one here, that a variable is "a generalization of patterns." Beginning algebra should be understood as generalized arithmetic. A letter such as "x" is used to represent only a number and nothing more. Computation with an expression in x is then the same as ordinary calculations with specific, familiar numbers. In this way, beginning algebra becomes a natural extension of arithmetic, as it should.

We are not arguing that standards calling upon students to recognize patterns should be eliminated. For example, it is desirable that children recognize patterns associated with even or odd numbers, be able to continue arithmetic and geometric sequences, and be able to express the nth terms of such sequences and others algebraically. Recognizing patterns can also aid in problem-solving or in posing conjectures. Our point here is that the attention given to patterns is excessive, sometimes destructive, and far out of balance with the actual importance of patterns in K-12 mathematics.

Manipulatives

Manipulatives are physical objects intended to serve as teaching aids. They can be helpful in introducing new concepts for elementary students, but too much use runs the risk that the students will focus on the manipulatives more than the mathematics, and even come to depend on them. Ultimately, the goal of elementary school math is to get students to manipulate numbers, not objects, in order to solve problems.

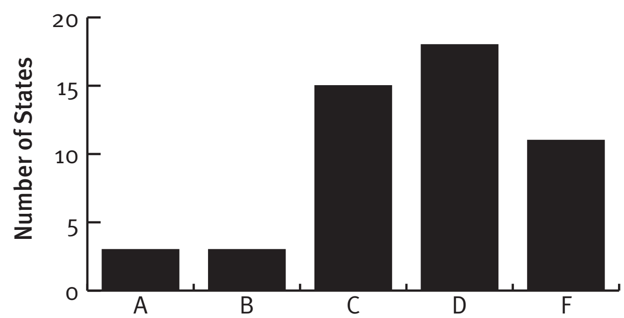
In higher grades, manipulatives can undermine important educational goals. There may be circumstances when a demonstration with a physical object is appropriate, but ultimately *paper and pencil are by far the most useful and important manipulatives.* They are the tools that students will use to do calculations for the rest of their lives. Mathematics by its very nature is abstract, and it is abstraction that gives mathematics its power.

Yet many state standards documents recommend and even require the use of a dizzying array of manipulatives for instruction or assessment in counterproductive ways. New Jersey's assessment requires that students be familiar with a collection of manipulatives that includes base ten blocks, cards, coins, geoboards, graph paper, multi-link cubes, number cubes (more commonly known as dice), pattern blocks, pentominoes, rulers, spinners, and tangrams. Kansas incorrectly refers to manipulatives as "Mathematical Models," and uses that phrase 572 times in its framework. The vast array of physical devices that Kansas math students must master includes place value mats, hundred charts, base ten blocks, unifix cubes, fraction strips, pattern blocks, geoboards, dot paper, tangrams, and attribute blocks. It is unclear in these cases whether students learn about manipulatives in order to better understand mathematics, or the other way around.

New Jersey and Kansas are far from unique in this regard. According to Alabama's introduction to its sixth-grade standards, "The sixth-grade curriculum is designed to maximize student learning through the use of manipulatives, social interaction, and technology." In New Hampshire, eighth-graders are required to "perform polynomial operations with manipulatives." Eighth-graders in Arkansas must "use manipulatives and computer technology (e.g., algebra tiles, two color counters, graphing calculators, balance scale model, etc.) to develop the concepts of equations."

The requirement to use algebra tiles in high school algebra courses is both widespread and misguided. Rather than requiring the use of plastic tiles to multiply and factor polynomials, states should insist that students become adept at using the distributive property, which is vastly more powerful and much simpler.

Figure 5: Final Grade Distribution, 2005



Estimation

Fostering estimation skills in students is a commendable goal shared by all state standards documents. However, there is a tendency to overemphasize estimation at the expense of exact arithmetic calculations. Idaho provides a useful illustration. Its first- and second-grade standards prematurely introduce estimation and "reasonableness" of results. These skills are more appropriately developed in the higher grades, after students have experience with exact calculations. In the elaboration of one first-grade standard, this example is provided: "Given *9 - 4*, would 10 be a reasonable number?" Similarly, for second grade, one finds: "Given subtraction problem, *38 - 6*, would 44 be a reasonable answer?" These examples are misguided. For these subtractions, the correct answer is the only reasonable answer. The notion of "reasonableness" might be addressed in grades 1 and 2 in connection with measurement, but not in connection with arithmetic of small whole numbers. Care should be taken not to substitute estimation for exact calculations.

Probability and Statistics

With few exceptions, state standards documents at all grade levels include strands of standards devoted to probability and statistics. Standards of this type almost invariably begin in Kindergarten (and sometimes pre-Kindergarten). Utah, for example, asks its Kindergartners to "understand basic concepts of probability," an impossible demand since probabilities are numbers between 0 and 1 and Kindergartners do not have a clear grasp of fractions. Perhaps in recognition of this, Utah's Kindergarten requirement includes the directive, "Relate past events to future events (e.g., The sun set about 6:00 last night, so it will set about the same time tonight)." But how such a realization about sunsets contributes to understanding basic concepts of probability is anyone's guess. Probability standards at the Kindergarten level are unavoidably ridiculous. In a similar vein, Vermont's first-graders are confronted with this standard:

For a probability event in which the sample space may or may not contain equally likely outcomes, use experimental probability to describe the likelihood or chance of an event (using "more likely," "less likely").

Again, this is premature and pointless. There is nothing to be gained by introducing the subject of probability to students who do not have the prerequisites to understand it. The state report cards that follow are full of similar examples.

Coherent mathematics standards delay the introduction of probability until middle school, and then proceed quickly by building on students' knowledge of fractions and ratios. Indiana does not have a probability and statistics strand for grades K-3. Other states would do well to emulate that commendable feature and carry it further by postponing most of their elementary school probability standards until middle school.

Many states also include data collection standards that are excessive. New York's third- and fourth-graders, for example, are required to:

Make predictions, using unbiased random samples.

- *Collect statistical data from newspapers, magazines, polls.*
- *Use spinners, drawing colored blocks from a bag, etc.*
- *Explore informally the conditions that must be checked in order to achieve an unbiased random sample (i.e., a set in which every member has an equal chance of being chosen) in data gathering and its practical use in television ratings, opinion polls, and marketing surveys.*

The time used for such open-ended activities would be better spent on mathematics.

Statistics and probability requirements typically appear with standards for all other mathematical topics, and often crowd out important topics in algebra and geometry. For example, West Virginia's Algebra I students are required to "perform a linear regression and use the results to predict specific values of a variable, and identify the equation for the line of regression," and to "use process (flow) charts and histograms, scatter diagrams, and normal distribution curves." Conflating geometry with statistics, Texas sixth-graders are required to "generate formulas to represent relationships involving perimeter, area, volume of a rectangular prism, etc., from a table of data." Statistical explorations should not replace a coherent geometric development of perimeter, area, and volume. Mississippi's Algebra II students "use scatter plots and apply regression analysis to data." While not always identified in the short state reports that follow, standards requiring visual estimation of lines or curves of best fit for statistical data are abundant in middle and high school algebra and geometry courses. Finding the coefficients for lines of best fit is college-level mathematics and is best explained at that level. The K-12 alternatives are to ask students to "eye ball" lines of best fit, or merely press calculator buttons without understanding what the machines are doing. Students would be better off learning, for example, rational function arithmetic, or how to complete the square for a quadratic polynomial—topics frequently missing or abridged.

Mathematical Reasoning and Problem-Solving

Problem solving is an indispensable part of learning mathematics and, ideally, straightforward practice problems should gradually give way to more difficult problems as students master skills. Unfortunately, few states offer standards that guide the development of problem-solving in a useful way. Students should solve single-step word problems in the earliest grades and deal with increasingly more challenging, multi-step problems as they progress.

As important as problem-solving is, there is much more to mathematical reasoning than solving word problems alone. Fordham I presents an illuminating discussion of mathematical reasoning in K-12 mathematics that includes this elaboration:

> *The beauty and efficacy of mathematics both derive from a common factor that distinguishes mathematics from the mere accretion of information, or application of practical skills and feats of memory. This distinguishing feature of mathematics might be called mathematical reasoning, reasoning that makes use of the structural organization by which the parts of mathematics are connected to each other, and not just to the real world objects of our experience, as when we employ mathematics to calculate some practical result.*[2]

The majority of states fail to incorporate mathematical reasoning directly into their content standards. Even for high school geometry, where it is difficult to avoid mathematical proofs, many state documents do not ask students to know proofs of anything in particular. Few states expect students to see a proof of the Pythagorean Theorem or any other theorem or any collection of theorems. Mathematical proofs should also be integrated into algebra and trigonometry courses, but it is a rare state that asks students even to know how to derive the quadratic formula in a high school algebra course.

Mathematical reasoning should be an integral part of the content at all grade levels. For example, elementary and middle school geometry standards should ask students to understand how to derive formulas for areas of simple figures. Students should be guided through a logical, coherent progression of formulas by relating areas of triangles to areas of rectangles, parallelograms, and trapezoids. But many states expect only that children will compute areas when given correct formulas. An example—one of many—is this North Dakota seventh-grade standard:

> *Students, when given the formulas, are able to find circumference, perimeter, and area of circles, parallelograms, triangles, and trapezoids (whole number measurements).*

Not only does this standard not ask for understanding of the basic area formulas, students aren't even asked to achieve the modest goal of memorizing them. We note also that the restriction in this standard to whole numbers is unnecessary and counterproductive at the seventh grade level, when knowledge of the arithmetic of real numbers, including pi, is clearly assumed in this very instruction.

Fig. 6: Changes in State Grades, 2000 - 2005

Big Improvement	Small Improvement	Same	Small Decline	Big Decline
IN	ID	AL	AZ	DE
LA	IL	AK	AR	DC
MA	MN	CA	CT	KS
MI	ND	CO	FL	MS
NM	TN	GA	KY	NH
		HI	NE	NC
		ME	NJ	OH
		MD	NY	SC
		MO	OK	SD
		MT	PA	UT
		NV	TX	
		OR	VT	
		RI	VA	
		WA	WV	
			WI	
			WY	

NOTE: Big improvement (or decline) signifies movement of more than one letter grade.

The logical development of fractions and decimals deserves special attention, rarely given in state documents. In many cases, students are inappropriately expected to multiply and divide decimal numbers a year in advance of multiplying and dividing fractions. This is problematic. What does it *mean* to multiply or divide

[2] *State Math Standards*, by Ralph Raimi and Lawrence Braden, Thomas B. Fordham Foundation, March 1998, page 9.

decimal numbers, if those operations for fractions have not been introduced? How are these operations defined? All too often, we found no indication that students should understand multiplication and division of rational numbers except as procedures.

In many cases, reliance on technology replaces mathematical reasoning. An example is this Ohio standard for seventh grade:

> *Describe differences between rational and irrational numbers; e.g., use technology to show that some numbers (rational) can be expressed as terminating or repeating decimals and others (irrational) as non-terminating and non-repeating decimals.*

The technology is not specified, but calculators cannot establish the fact that rational numbers necessarily have repeating or terminating decimals. On the other hand, the characterization of decimal expansions of rational numbers can be made in a straightforward manner using the long division algorithm.

Mathematical reasoning is systematically undermined when prerequisites for content standards are insufficiently developed. When arithmetic, particularly fraction arithmetic, is poorly developed in the elementary grades, students have little hope of understanding algebra as anything other than a maze of complicated recipes to be memorized, as is too often the case in state standards documents.

Perhaps the most strident denial of the importance of prerequisites in mathematics appears in Hawaii's Framework:

> *Learning higher-level mathematics concepts and processes are [sic] not necessarily dependent upon "prerequisite" knowledge and skills. The traditional notion that students cannot learn concepts from Algebra and above (higher-level course content) if they don't have the basic skill operations of addition, subtraction, etc. has been contradicted by evidence to the contrary.*

Unsurprisingly, no such evidence is cited for this wrong headed assertion. Prerequisites cannot be discarded. They are essential to mathematics. The failure to develop appropriate prerequisites and mathematical reasoning based on those prerequisites leads to the degeneration of mathematics standards into what might be described as mathematics appreciation. Hawaii is part of an unfortunate trend among the states to introduce calculus concepts too early and without necessary prerequisites. Thus, Hawaiian fourth graders are asked to identify and describe "situations with varying rates of change such as time and distance [*sic*]." Likewise, with no development of calculus prerequisites, one of Maryland's algebra standards is:

> *The student will describe the graph of a non-linear function and discuss its appearance in terms of the basic concepts of maxima and minima, zeros (roots), rate of change, domain and range, and continuity.*

Pennsylvania's Framework even has a strand entitled "Concepts of Calculus," which lists standards for each of the grades 3, 5, 8, and 11. Fifth-graders are supposed to "identify maximum and minimum." This directive is given without specifying the type of quantity for which extrema are to be found, or any method to carry out such a task. Pennsylvania's eleventh-grade standards under this strand also have little substance. Without any mention of limits, derivatives, or integrals, and no further elaboration, they require students to "determine maximum and minimum values of a function over a specified interval" and "graph and interpret rates of growth/decay."

Similarly out of place and unsupported by any discussion of derivatives is the South Carolina Algebra II standard: "Determine changes in slope relative to the changes in the independent variable." But perhaps the most bizarre of what might be termed "illusory calculus" standards is this New Mexico grade 9-12 standard:

> *Work with composition of functions (e. g., find f of g when* $f(x) = 2x - 3$ *and* $g(x) = 3x - 2$*), and find the domain, range, intercepts, zeros, and local maxima or minima of the final function.*

We note that there is no hint of calculus in any of the New Mexico grade 9-12 standards except for this one. Further, why restrict the identification of local extreme values only to compositions of functions? Compounding the confusion, since these two functions *f(x)* and *g(x)* are linear, their composition is also linear, and there are no maximum or minimum values of that composition.

The failure to fully recognize prerequisites as essential to learning mathematics not only leads to premature coverage of calculus topics, but opens the floodgates for superficial content standards. For example, a Missouri standard (under the heading of "What All Students Should Be Able To Do") absurdly asks high school students to,

> *Evaluate the logic and aesthetics of mathematics as they relate to the universe.*

Similar examples of inflation appear in many state standards.[3]

The Roots of, and Remedy for, Bad Standards

Why are so many state standards documents of such low quality? What factors influence their content? What accounts for the uniformity of their flaws?

The National Council of Teachers of Mathematics (NCTM) has had, and continues to have, immense influence on state education departments and K-12 mathematics education in general. Many state standards adhere closely to guidelines published by the NCTM in a long sequence of documents. Three have been especially influential: *An Agenda for Action* (1980), *Curriculum and Evaluation Standards for School Mathematics* (1989), and *Principles and Standards for School Mathematics* (2000). We refer to the latter two documents respectively as the 1989 NCTM Standards and the 2000 NCTM Standards.

An Agenda for Action was the blueprint for the later documents, paving the way for current trends when it called for "decreased emphasis on such activities as . . . performing paper-and-pencil calculations with numbers of more than two digits." This would be possible, the document explained, because "the use of calculators has radically reduced the demand for some paper-and-pencil techniques." Accordingly, "all students should have access to calculators and increasingly to computers throughout their school mathematics program." This includes calculators "for use in elementary and secondary school classrooms." Regarding basic skills, the report warned, "It is dangerous to assume that skills from one era will suffice for another." *An Agenda for Action* further stressed that "difficulty with paper-and-pencil computation should not interfere with the learning of problem-solving strategies." Foreshadowing another trend among state standards documents, the 1980 report also encouraged "the use of manipulatives, where suited, to illustrate or develop a concept or skill."

Fig. 7: Changes in State Grades, 1998 - 2005

Big Improvement	Small Improvement	Same	Small Decline	Big Decline
IN	ID	AL	AK	DE
LA	IL	AR	AZ	MS
MD	ME	CA	CT	NH
MA	NE	CO	FL	NC
MI	ND	DC	KY	OH
NM		GA	NJ	UT
OK		HI	NY	
SD		MO	TN	
		OR	TX	
		PA	VT	
		RI	VA	
		SC	WV	
		WA	WI	

NOTE: Big improvement (or decline) signifies movement of more than one letter grade.

The 1989 NCTM Standards amplified and expanded *An Agenda for Action*. It called for some topics to receive increased attention in schools and other topics to receive decreased attention. Among the grade K-4 topics slated for greater attention were "mental computation," "use of calculators for complex computation," "collection and organization of data," "pattern recognition and description," and "use of manipulative materi-

[3] "Inflation" is one of two subcategories of the "negative qualities" criterion used in the evaluation of standards documents. See the section, *Criteria for Evaluation,* page 31.

als." The list of topics recommended for decreased attention included "complex paper-and-pencil computations," "long division," "paper and pencil fraction computation," "rote practice," "rote memorization of rules," and "teaching by telling." For grades 5-8, the 1989 NCTM Standards took an even more radical position, recommending for de-emphasis "manipulating symbols," "memorizing rules and algorithms," "practicing tedious paper-and-pencil computations," and "finding exact forms of answers."

Like *An Agenda for Action*, the 1989 NCTM Standards put heavy emphasis on calculator use at all grade levels. On page 8, it proclaimed, "The new technology not only has made calculations and graphing easier, it has changed the very nature of mathematics" and recommended that "appropriate calculators should be available to all students at all times."

The influence of the 1989 NCTM Standards on state standards can hardly be overstated. After the publication of Fordham I, author Ralph Raimi wrote:

> *These state standards, though federally encouraged and supported, are supposed to be each state's vision of the future, of what mathematics education ought to be. Some were apparently written by enormous committees of teachers and math education specialists, but the final texts obviously were assembled and organized at the state education department level sometimes with the help of one of the regional educational "laboratories" set up and financed by the U.S. Department of Education. Despite the regional differences, the influence of NCTM and these laboratories has imparted a certain sameness to many of the state standards we ended up studying. Almost all of them had publication dates of 1996 or 1997.*[4]

Many of the documents evaluated in this Fordham report were also published, or drafted, prior to the appearance of the 2000 NCTM Standards.

The 1989 NCTM Standards document was the subject of harsh criticism during the 1990s. As a consequence, some of the more radical declarations of the 1989 document were eliminated in the revised 2000 NCTM Standards. However, the latter document promoted the same themes of its predecessors, including emphasis on calculators, patterns, manipulatives, estimation, nonstandard algorithms, etc. Much of the sameness of current state standards documents may be traced to the NCTM's vision of mathematics education.

A fuller explanation for the shortcomings of state math standards, however, goes beyond the influence of the NCTM and takes into account the deficient mathematical knowledge of many state standards authors. Mathematical ignorance among standards writers is the greatest impediment to improvement.

Some guidelines for improving standards, based on this report, suggest themselves immediately. States can correct the "common problems" identified in this essay, such as overuse of calculators and manipulatives, overemphasis of patterns and probability and statistics, and insufficient development of the standard algorithms of arithmetic and fraction arithmetic. But here the devil is in the details and these corrections should not be attempted by the people who created the problems in the first place. For the purpose of writing standards, there is no substitute for a thorough understanding of mathematics—not mathematics education or pedagogy, but the subject matter itself. A state education department's usual choice of experts for this task will likely cause as many new problems as it solves.

Of particular importance is a coherent and thorough development of arithmetic in the early grades, both in terms of conceptual understanding *and* computational fluency. Without a solid foundation in this most important branch of mathematics—arithmetic—success in secondary school algebra, geometry, trigonometry, and pre-calculus is impossible. The challenges in developing credible arithmetic standards should not be underestimated. Standards authors lacking a deep understanding of mathematics, including advanced topics, are not up to the task.

A simple and effective way to improve standards is to adopt those of one of the top scoring states: California, Indiana, or Massachusetts. At the time of this writing,

[4] "Judging State Standards for K-12," by Ralph Raimi, Chapter 2 in *What's at Stake in the K-12 Standards Wars: A Primer for Educational Policy Makers*, edited by Sandra Stotsky, Peter Lang Publishing, page 40.

the District of Columbia was considering replacing its standards with the high quality standards from one of these states. That makes good sense. There is no need to reinvent the wheel. The goal of standards should not be innovation for its own sake; the goal is to implement useful, high-quality standards, regardless of where they originated.

Four Antidotes to Faulty State Standards

1. **Replace the authors of low-quality standards documents with people who thoroughly understand the subject of mathematics. Include university professors from mathematics departments.**

2. **Develop coherent arithmetic standards that emphasize both conceptual understanding *and* computational fluency.**

3. **Avoid the "common problems" described above, such as overuse of calculators and manipulatives, overemphasis of patterns and probability and statistics, and insufficient development of the standard algorithms of arithmetic and fraction arithmetic.**

4. **Consider adopting a complete set of high-quality math standards from one of the top scoring states: California, Indiana, or Massachusetts.**

If, however, a state chooses to develop its own standards in whole or in part, some university level mathematicians (as distinguished from education faculty) should be appointed to standards writing committees and be given enough authority over the process so that their judgments cannot easily be overturned. Such a process was used in California in December 1997 and resulted in the highest-ranked standards in all three Fordham math standards evaluations. The participation of university math professors in the development of K-12 standards is becoming increasingly important. Since 1990, more than 60 percent of high school graduates have gone directly to colleges and universities[5] and that percentage is likely to increase. College preparation should therefore be the default choice (though not the only option) for K-12 mathematics. For this purpose, the perspective of university mathematics professors on what is needed in K-12 mathematics to succeed in college is indispensible.

[5] National Center for Education Statistics, Table 183 – College enrollment rates of high school completers, by race/ethnicity: 1960 to 2001.

Memo to Policy Makers

Justin Torres

What are we to think of the state of K-12 math standards across the U.S. in 2005? More to the point, what should governors, legislators, superintendents, school board members, instructional leaders—the legions of policy makers who affect curricular and instructional choices in states and districts—make of David Klein's provocative findings? What should they do to improve matters?

Both Klein (at page 13) and Chester Finn (see Foreword, page 5) provide important insights. Finn sets the policy scene, tracing the history of standards development up to the present, when No Child Left Behind is beginning to drive state standards and accountability policies and the Bush administration seeks to extend this regimen to the high school. Klein enumerates problems that are depressingly common in today's state math standards and shows how both the National Council of Teachers of Mathematics and the composition of standards-writing committees have contributed to math standards that, in most jurisdictions, continue to fall woefully short of what's needed.

What Can Policy Makers Do?

One of Klein's recommendation makes immediate sense: States should consider adopting or closely emulating the standards of one of the top scoring states: California, Indiana, or Massachusetts. At the time of this writing, the District of Columbia was considering replacing its standards with the high-quality standards from Massachusetts. As Klein says, "There is no need to reinvent the wheel. The goal of standards should not be innovation for its own sake; the goal is to implement useful, high quality standards, regardless of where they originated." Kudos to new D.C. superintendent Clifford Janey for grasping this point and acting in the best interests of District schoolchildren.

Yet we know that many states will continue to draft their own standards, for a variety of reasons. And so we want to provide them with some practical guidance on how to develop K-12 math standards that make preparation for college and the modern workforce the "default" track for today's elementary/secondary students.

Why should standards-writers be concerned? As Klein points out, increasing numbers of American high school students are going on to college. Indeed, it's fair to say that nearly all of tomorrow's high school graduates will sooner or later have some exposure to post-secondary education. They'd best be ready for it.

Yet many higher education institutions report that increasing numbers of entering students—even at selective campuses—require remedial mathematics education. (At California State University, where Klein himself teaches, that number now tops 50 percent, while in some community colleges it approaches two-thirds of all entering students.) The cost to society of this remedial effort is tremendous, both directly to colleges forced to teach skills that should have been learned in middle and high schools, and indirectly through lost productivity, workplace error, and the defensive measures that innumerable institutions must now take to combat the ignorance of their employees, citizens, taxpayers, neighbors, etc.

One study, from April 2004, attempted to count the direct and indirect costs of remedial education in just one state, Alabama. The findings ranged from $304 million to $1.17 billion per year, with a best estimate of $541 million annually—again, in a single state. Businesses, the report concluded, had a difficult time finding employees who had adequate math and writing skills. The president of a temporary staffing firm wrote to the study's authors to note the large number of entry-level applicants who do not know how many inches are in a foot.[6]

[6] *The Cost of Remedial Education: How Much Alabama Pays When Students Fail to Learn Basic Skills*, by Christopher W. Hammons, Alabama Policy Institute, 2004, page 9.

Nor is remediation itself the only “cost” of inadequate pre-college education in core fields such as mathematics. Billions of student aid dollars are, in effect, wasted every year by being expended on the education of people who drop out, flunk out, or give up on higher education when they realize that they’re not prepared for it. And then there’s the immense cost in human potential, wasted time, unfulfilled dreams, and dashed hopes.

Consider, too, the implications for American society and its economy as the qualifications of our workforce slip further and further behind those of other lands. See, for example, the new evidence from the quadrennial Program for International Student Assessment (PISA): The math skills of American 15-year-olds are sub-standard and falling, compared to their international peers. In fact, the U.S. is outperformed by almost every developed nation, beating only poorer countries such as Mexico and Portugal. This is depressing enough, but if you look closely at the results, things get worse. The achievement gap between whites and minorities persists, and a full one-quarter of American students performed at the lowest possible level of competence *or below*—meaning they are unable to perform the simplest calculations.

Recent results from the Trends in International Mathematics and Science Study (TIMMS) are better but still cause for concern. U.S. students lag behind a number of European and Asian nations in math performance, and fourth-grade scores barely moved since 1999. (Scores for eighth-graders improved.) Only 7 percent of young Americans scored at the “advanced” level on TIMMS, versus 44 percent in Singapore and 38 percent in Taiwan.

If American schoolchildren can’t keep up with their international peers, one obvious consequence is the outsourcing of skilled jobs to other lands, with all its consequences for unemployment on these shores. Federal Reserve chairman Alan Greenspan made the same point in March 2004 in a speech that called for better math and science education as both a defense against and a solution to job outsourcing. “The capacity of workers, after being displaced, to find a new job that will eventually provide nearly comparable pay most often depends on the general knowledge of the worker and the ability of that individual to learn new skills,” he noted.

Raising the Bar

One important insight was supplied in February 2004 by the American Diploma Project (www.achieve.org), whose analysts found that colleges and modern employers converge around the skills and knowledge needed by high school graduates (in math especially) for success in both higher education and the modern workplace. (Achieve has also done valuable work setting benchmarks for state math standards aligned to these “exit” expectations, and evaluating states against them.) Put simply: What young people need to know and be able to do to succeed in higher education is essentially the same as what they need to succeed in tomorrow’s jobs. Thus it makes enormous sense for all high schoolers to master these common, foundational skills. The fact that many students don’t is due in no small part to the fact that states don’t set the bar high enough in their state standards and tests, especially their high school exit exams.

Instead, many state standards documents cover a variety of topics in a disconnected manner, with no organizing principle to guide expectations and instruction in K-12 mathematics. Constructing standards with college preparation in mind would provide both a framework for coherence in the standards themselves and criteria for choosing which topics should be emphasized and which can be given less attention. Knowing where you’re going when developing a set of math standards makes it easier to determine which steps to take along the way. In other words, if you know where you want twelfth-graders to end up by way of knowledge and skills, you can “backward map” all the way to Kindergarten to ensure that the necessary teaching-and-learning steps get taken in the appropriate sequence.

The first step, of course, is mastery of arithmetic in the elementary grades. Without it, there’s no hope of ADP-level or college-prep level math being mastered in high school. It says something deeply unsettling about the parlous state of math education in these United States that the arithmetic point must even be raised—but it must. As Klein notes, “Without a solid foundation in this most important branch of mathematics—arithmetic—success in secondary school algebra, geometry, trigonometry, and pre-calculus is impossible.” This failure, then, is profoundly consequential. Standards-writers guided by the goal of immersing all students in college-level mathematics need to work back through the grades to develop the skills at the appropriate pace and level of difficulty. That mapping must reach all the way back to the most elementary topic in mathematics—arithmetic—and to a child's first exposure to arithmetic in Kindergarten and the primary grades.

The results of David Klein's evaluation of state math standards show that there is clearly much to be done in setting high standards and ensuring that every child meets them. It is painstaking—but deeply necessary—work that, to be successful, requires clear goals, competent standards-writers, and a willingness to face hard truths about what is needed to prepare students for higher education and productive employment. And it is work that, even in the results-driven era of No Child Left Behind, has only just begun.

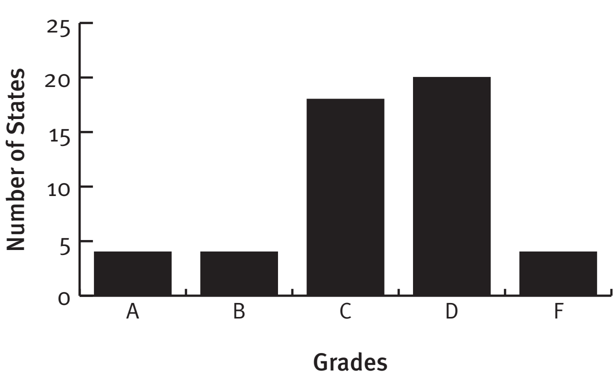
Justin Torres
Research Director
Washington, D.C.
January 2005

Criteria for Evaluation

State standards were judged on a 0-4 point scale on four criteria: clarity, content, reason, and negative qualities. In each case, 4 indicates excellent performance, 3 indicates good performance, 2 indicates mediocre performance, 1 indicates poor performance, and 0 indicates failing performance. More information about how grades were assigned is available in the "Methods and Procedures" section beginning on page 121.[7]

Clarity

Fig. 8: 2005 Grades for Clarity



State average: 1.85
Range: 0.33-3.83

States to watch:
California (3.83)
Indiana, Massachusetts (3.67)
Georgia (3.33)
Alabama, New Mexico (3.00)

States to shun:
Washington, Connecticut (0.33)
Missouri (0.67)
Delaware (0.83)

Clarity refers to the success the document has in achieving its own purpose, i.e., making clear to teachers, test developers, textbooks authors, and parents what the state desires. Clarity refers to more than the prose, however. The clarity grade is the average of three separate sub-categories:

1. *Clarity* of the language: The words and sentences themselves must be understandable, syntactically unambiguous, and without needless jargon.

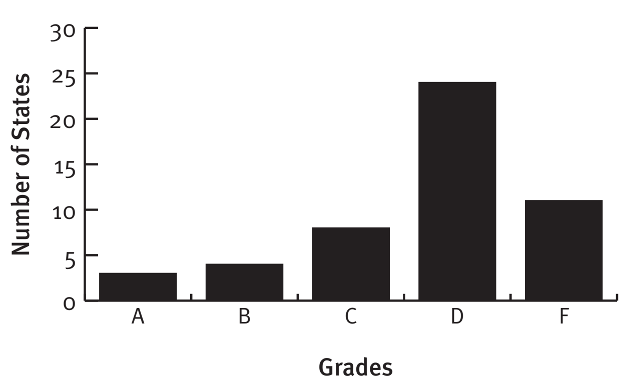
2. *Definiteness* of the prescriptions given: What the language says should be mathematically and pedagogically definite, leaving no doubt of what the inner and outer boundaries are, of what is being asked of the student or teacher.

3. *Testability* of the lessons as described: The statement or demand, even if understandable and completely defined, might yet ask for results impossible to test in the school environment. We assign a positive value to testability.

For comparisons of clarity grades between the three Fordham Foundation math standards evaluations, see the Appendix beginning on page 123.

Content

Fig. 9: 2005 Grades for Content



[7] Much of this section is adapted from the "Criteria for Evaluation" section of *State Math Standards*, by Ralph A. Raimi and Lawrence Braden, Thomas B. Fordham Foundation, March 1998.

State average: 1.57
Range: 0.33-3.94

States to watch:
California (3.94)
Indiana (3.83)
Massachusetts (3.67)
Alabama (3.17)

States to shun:
Connecticut, Hawaii, Missouri (0.33)

Content, the second criterion, is plain enough in intent. Mainly, it is a matter of what might be called "subject coverage," i.e., whether the topics offered and the performance demanded at each level are sufficient and suitable. To the degree we can determine it from the standards documents, we ask, is the state asking K-12 students to learn the correct skills, in the best order and at the proper speed? For this report, the content score comprises 40 percent of the total grade for any state.

Here we separate the curriculum into three parts (albeit with fuzzy edges): Primary, Middle, and Secondary. It is common for states to offer more than one 9-12 curriculum, but also to print standards describing only the "common" curriculum, often the one intended for a universal graduation exam, usually in grade 11.

We cannot judge the division of content with year-by-year precision because few states do so, and we wish our scores to be comparable across states. As for the fuzziness of the edges of the three grade-span divisions, not even all those states with "elementary," "intermediate," and "high school" categories divide in the same way. One popular scheme is K-6, 7-9, and 10-12, while others divide it K-5, 6-8, and 9-12. In cases where states divide their standards into many levels (sometimes year-by-year), we shall use the first of these schemes. In other cases we accept the state's divisions and grade accordingly. Therefore, Primary, Middle, and Secondary will not necessarily mean the same thing from one state to another. There is really no need for such precision in our grading, though of course in any given curriculum it does make a difference where topics are placed.

Content gives rise to three criteria:

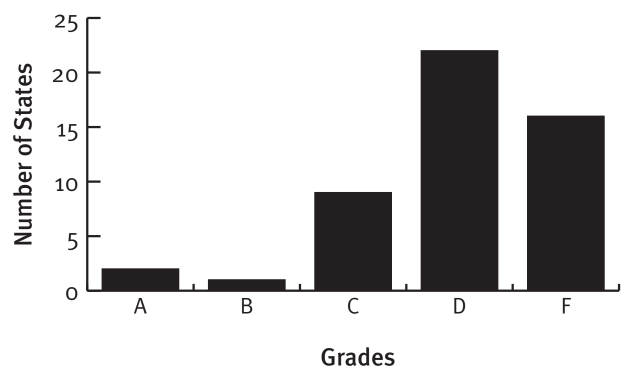
1. Primary school content (K-5, approximately)
2. Middle school content (or 6-8, approximately)
3. Secondary school content (or 9-12, approximately).

In many states, mathematics is mandatory through the tenth grade, while others might vary by a year or so. Our judgment of the published standards does not take account of what is or is not mandatory; thus, a rating will be given for secondary school content whether or not all students in fact are exposed to part or all of it. (Some standards documents only describe the curriculum through grade 11, and we adjust our expectations of content accordingly.)

For comparisons of content grades between the three Fordham Foundation math standards evaluations, see the Appendix beginning on page 123.

Reason

Fig. 10: 2005 Grades for Reason



State average: 1.15
Range: 0.00-4.00

States to watch:
Indiana (4.00)
California (3.83)
West Virginia (3.00)

States to shun:
Arkansas, Connecticut, Hawaii, Montana, New Hampshire, Oregon, Rhode Island, Wyoming (0.00)

Civilized people have always recognized mathematics as an integral part of their cultural heritage. Mathematics is the oldest and most universal part of our culture. In fact, we share it with all the world, and it has its roots in the most ancient of times and the most distant of lands.

The beauty and efficacy of mathematics derive from a common factor that distinguishes mathematics from the mere accretion of information, or application of practical skills and feats of memory. This distinguishing feature of mathematics may be called *mathematical reasoning*, reasoning that makes use of the structural organization by which the parts of mathematics are connected to each other, and not just to the real-world objects of our experience, as when we employ mathematics to calculate some practical result.

The essence of mathematics is its *coherent quality*. Knowledge of one part of a logical structure entails consequences that are inescapable and can be found out by reason alone. It is the ability to *deduce consequences* that would otherwise require tedious observation and disconnected experiences to discover, which makes mathematics so valuable in practice; only a confident command of the method by which such deductions are made can bring one the benefit of more than its most trivial results.

Should this coherence of mathematics be inculcated in the schools, or should it be confined to professional study in the universities? A 1997 report from a task force formed by the Mathematical Association of America to advise the National Council of Teachers of Mathematics in its revision of the 1989 NCTM Standards argues for its early teaching:

> *[T]he foundation of mathematics is reasoning. While science verifies through observation, mathematics verifies through logical reasoning. Thus the essence of mathematics lies in proofs, and the distinction among illustrations, conjectures and proofs should be emphasized. . . .*
>
> *If reasoning ability is not developed in the students, then mathematics simply becomes a matter of following a set of procedures and mimicking examples without thought as to why they make sense.*

Even a small child should understand how the memorization of tables of addition and multiplication for the small numbers (1 through 10) necessarily produces all other information on sums and products of numbers of any size whatever, once the structural features of the decimal system of notation are fathomed and applied. At a more advanced level, the knowledge of a handful of facts of Euclidean geometry—the famous Axioms and Postulates of Euclid, or an equivalent system—necessarily implies (for example) the useful Pythagorean Theorem, the trigonometric Law of Cosines, and a tower of truths beyond.

Any program of mathematics teaching that slights these interconnections doesn't just deprive the student of the beauty of the subject, or his appreciation of its philosophic import in the universal culture of humanity, but even at the practical level it burdens that child with the apparent need for memorizing large numbers of disconnected facts, where reason would have smoothed his path and lightened his burden. People untaught in mathematical reasoning are not being saved from something difficult; they are, rather, being deprived of something easy.

Therefore, in judging standards documents for school mathematics, we look to the "topics" as listed in the "content" criteria not only for their sufficiency, clarity, and relevance, but also for whether their statement includes or implies that they are to be taught with the explicit inclusion of information on their standing within the overall structures of mathematical reason.

A state's standards will not score higher on the Reason criterion just by containing a thread named "reasoning," "interconnections," or the like. It is, in fact, unfortunate that so many of the standards documents contain a thread called "Problem-solving and Mathematical Reasoning," since that category often slights the reasoning in favor of the "problem-solving," or implies that they are essentially the same thing. Mathematical reasoning is not found in the connection between mathematics and the "real world," but in the logical interconnections within mathematics itself.

Since children cannot be taught from the beginning "how to prove things" in general, they must begin with experience and facts until, with time, the interconnections of facts manifest themselves and become a subject of discussion, with a vocabulary appropriate to the level. Children must then learn how to prove certain particular things, memorable things, both as examples for reasoning and for the results obtained. The quadratic formula, the volume of a prism, and why the angles of a triangle add to a straight angle, are examples. What does the distributive law have to do with "long multiplication?" Why do independent events have probabilities that combine multiplicatively? Why is the product of two numbers equal to the product of their negatives?

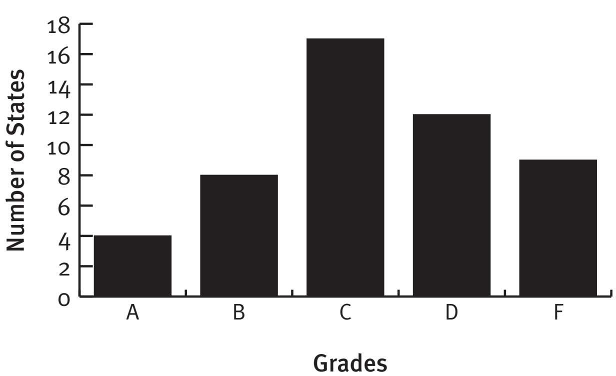
(At a more advanced level, the reasoning process can itself become an object of contemplation; but except for the vocabulary and ideas needed for daily mathematical use, the study of formal logic and set theory are not for K-12 classrooms.)

We therefore look at the *standards documents as a whole* to determine how well the subject matter is presented in an order, wording, or context that can only be satisfied by including due attention to this most essential feature of all mathematics.

For comparisons of reason grades between the three Fordham Foundation math standards evaluations, see the Appendix beginning on page 123.

Negative Qualities

Fig. 11: 2005 Grades for Negative Qualities



State average: 1.79
Range: 0.00-3.92

States to watch:
California (3.92)
Indiana (3.75)
Alabama, Massachusetts (3.50)
New Mexico, North Dakota (3.00)

States to shun:
Delaware, Washington (0.00)
Kansas (0.25)
Florida, Hawaii, Missouri (0.50)

This fourth criterion looks for the presence of unfortunate features of the document that contradict its intent or would cause its reader to deviate from what otherwise good, clear advice the document contains. We call one form of it *False Doctrine*. The second form is called *Inflation* because it offends the reader with useless verbiage, conveying no useful information. Scores for Negative Qualities are assigned a positive value; that is, a high score indicates the lack of such qualities.

Under False Doctrine, which can be either curricular or pedagogical, is whatever text contained in the standards we judge to be injurious to the correct transmission of mathematical information. To be sure, such judgments can only be our own, as there are disagreements among experts on some of these matters. Indeed, our choice of the term "false doctrine" for this category of our study is a half-humorous reference to its theological origins, where it is a synonym for heresy. Mathematics education has no official heresies, of course; yet if one must make a judgment about whether a teaching ("doctrine") is to be honored or marked down, deciding whether an expressed doctrine is true or false is necessary.

The NCTM, for example, prescribes the early use of calculators with an enthusiasm the authors of this report deplore, and the NCTM discourages the memorization of certain elementary processes, such as "long division" of decimally expressed real numbers, and the paper-and-pencil arithmetic of all fractions, that we think essential. We assure the reader, however, that our view is not merely idiosyncratic, but also has standing in the world of mathematics education.

While in general we expect standards to leave pedagogical decisions to teachers (as most standards documents do), so that pedagogy is not ordinarily something we rate in this study, some standards contain pedagogical advice that we believe undermines what the document otherwise recommends. Advice against memorization of certain algorithms, or a pedagogical standard mandating the use of calculators to a degree we consider mistaken, might appear under a pedagogical rubric. Then our practice of not judging pedagogical advice fails, for if the pedagogical part of the document gives advice making it impossible for the curricular part–as expressed there–to be accomplished properly, we must take note of the contradiction under this rubric of False Doctrine.

Two other false doctrines are excessive emphases on "real-world problems" as the main legitimating motive of mathematics instruction, and the equally fashionable notion that a mathematical question may have a multitude of different valid answers. Excessive emphasis on the "real-world" leads to tedious exercises in measuring playgrounds and taking census data, under headings like "Geometry" and "Statistics," in place of teaching mathematics. The idea that a mathematical question may have various answers derives from confusing a practical problem (whether to spend tax dollars on a recycling plant or a highway) with a mathematical question whose solution might form part of such an investigation. As the Mathematics Association of America Task Force on the NCTM Standards has noted,

> *[R]esults in mathematics follow from hypotheses, which may be implicit or explicit. Although there may be many routes to a solution, based on the hypotheses, there is but one correct answer in mathematics. It may have many components, or it may be nonexistent if the assumptions are inconsistent, but the answer does not change unless the hypotheses change.*

Constructivism, a pedagogical stance common today, has led many states to advise exercises in having children "discover" mathematical facts, algorithms, or "strategies." Such a mode of teaching has its value, in causing students to better internalize what they have learned; but wholesale application of this point of view can lead to such absurdities as classroom exercises in "discovering" what are really conventions and definitions, things that *cannot* be discovered by reason and discussion, but are arbitrary and must simply be learned.

Students are also sometimes urged to discover truths that took humanity many centuries to elucidate, such as the Pythagorean Theorem. Such "discoveries" are impossible in school, of course. Teachers so instructed will waste time, and end by conveying a mistaken impression of the standing of the information they must surreptitiously feed their students if the lesson is to come to closure. And often it all remains open-ended, confusing the lesson itself. Any doctrine tending to say that telling things to students robs them of the delight of discovery must be carefully hedged about with pedagogical information if it is not to be false doctrine, and unfortunately such doctrine is so easily and so often given injudiciously and taken injuriously that we deplore even its mention.

Finally, under False Doctrine must be listed the occurrence of plain mathematical error. Sad to say, several of the standards documents contain mathematical misstatements that are not mere misprints or the consequence of momentary inattention, but betray genuine ignorance.

Under the other negative rubric, *Inflation*, we speak more of prose than content. Evidence of mathematical ignorance on the part of the authors is a negative feature, whether or not the document shows the effect of this ignorance in its actual prescriptions, or contains outright mathematical error. Repetitiousness, bureaucratic jargon, or other evils of prose style that might cause potential readers to stop reading or paying attention, can render the document less effective than it should be, even if its clarity is not literally affected. Irrelevancies, such as the smuggling in of trendy political or social doctrines, can injure the value of a standards document by distracting the reader, even if they do not otherwise change what the standard essentially prescribes.

The most common symptom of irrelevancy, or evidence of ignorance or inattention, is bloated prose, the making of pretentious yet empty pronouncements. Bad writing in this sense is a notable defect in the collection of standards we have studied.

We thus distinguish two essentially different failures subsumed by this description of pitfalls, two Negative Qualities that might injure a standards document in ways not classifiable under the headings of Clarity and Content: Inflation (in the writing), which is impossible to make use of; and False Doctrine, which can be used but shouldn't.

For comparisons of Negative Qualities grades between the three Fordham Foundation math standards evaluations, see the Appendix beginning on page 123.

State Reports 2005

Scale Used for Converting a **Weighted Score** to a Letter Grade
3.25-4.0 = A
2.50-3.24 = B
1.75-2.49 = C
1.00-1.74 = D
0.00-0.99 = F

ALABAMA

Reviewed: *Alabama Course of Study: Mathematics*, 2003. Alabama provides grade-level standards for each of the grades K-8, Algebra I standards, and Geometry standards intended for almost all students. Following the geometry course, the *Alabama Course of Study: Mathematics* provides standards for a number of different courses of study to "accommodate the needs of all students" that include Algebraic Connections, Algebra II, Algebra II with Trigonometry, Algebra III with Statistics, and Precalculus.

2005 STATE REPORT CARD

Alabama

Clarity: **3.00**	B	
Content: **3.17**	B	
Reason: **2.00**	C	
Negative Qualities: **3.50**	B	
Weighted Score: 2.97	Final Grade:	B
2000 Grade: **B**		
1998 Grade: **B**		

Alabama's standards, revised in 2003, remain solid. They are clearly written and address the important topics. Students are expected to demonstrate "computational fluency," solve word problems, learn algebraic skills and ideas, and solve geometry problems, including some exposure to proofs. At each grade level, the standards include introductory remarks, with exhortations to "maximize student learning through the use of manipulatives, social interaction, and technology," as the sixth grade curriculum puts it. Though this statement overemphasizes the role of manipulatives and technology, except for such introductory remarks, calculators and technology are not mentioned in the standards themselves until ninth grade. Taken at face value, this policy of minimal calculator use is commendable.

More Memorization, Less Probability and Data Analysis

A weakness of the standards is that memorization of the basic number facts is not required. Instead, second-graders are expected to demonstrate "computational fluency for basic addition and subtraction facts with sums through eighteen and differences with minuends through eighteen, using horizontal and vertical forms." Similar language for the single-digit multiplication facts and corresponding division facts appears in the fourth grade standards. Computational fluency in determining the value of *9 x 7* is not the same as memorizing the basic arithmetic facts, which should be explicitly required of elementary grade students. Standard arithmetic algorithms, including the long division algorithm, are not mentioned in Alabama's standards, an inexplicable omission.

Probability and data analysis standards are overemphasized, appearing at every grade level and for every course. Second-graders are prematurely expected to "determine if one event related to everyday life is more likely or less likely to occur than another event." Third-graders are expected to

Determine the likelihood of different outcomes in a simple experiment.

Example: determining that the spinner is least likely to land on red in this diagram.

As the probability of any event is a number between 0 and 1, it makes no sense to discuss probability until students have at least a working knowledge of fractions.

Some of the standards relating to patterns are defective. For example, sixth-graders are expected to "solve problems using numeric and geometric patterns" by, for example, "continuing a pattern for the 5th and 6th numbers when given the first four numbers in the pattern." This is an example of false doctrine, since without a specific rule for the pattern, there are no correct or incorrect answers for such a problem.

The following standards regarding lines of best fit for scatter plots are given for eighth grade, Algebra I, and Geometry respectively:

Making predictions by estimating the line of best fit from a scatterplot.

Use a scatterplot and its line of best fit or a specific line graph to determine the relationship existing between two sets of data, including positive, negative, or no relationship.

Collect data and create a scatterplot comparing the perimeter and area of various rectangles. Determine whether a line of best fit can be drawn.

To develop the topic of lines of best fit properly is college-level mathematics, and to do it other ways is not mathematics.

The ubiquitous data analysis and probability standards weaken the high school course standards. Algebra I students would be better off learning to complete the square for quadratic polynomials—a topic not listed in the Algebra I standards—rather than trying to "eyeball" lines of best fit, or pressing calculator buttons without understanding what the machine is doing. Similar comments apply to the Geometry and higher-level course standards.

Alaska

Reviewed: *Alaska Content Standards*, 1999; *Alaska Performance Standards*, January 20, 1999; *Math Grade Level Expectations for Grades 3-10*, March 16, 2004. The *Content Standards* consist of general standards addressed uniformly to students in all grades, such as "use computational methods and appropriate technology as problem-solving tools." The more specific *Performance Standards* provides standards for students in four broad age bands, and *Grade Level Expectations* has detailed grade-level standards for each of the grades three to ten.

2005 STATE REPORT CARD

Alaska

Clarity: 2.00	C
Content: 1.17	D
Reason: 0.50	F
Negative Qualities: 1.75	C
Weighted Score: 1.32	Final Grade: D

2000 Grade: **D**

1998 Grade: **C**

In the elementary grades, students are expected to memorize the basic number facts, a positive feature, and are appropriately expected to be able to compute with whole numbers. But there is no mention of the standard algorithms; rather, the Performance Standards call upon students to "add and subtract . . . using a variety of models and algorithms." The Grade Level Expectations introduce calculators in third grade, far too early:

The student determines reasonable answers to real-life situations, paper/ pencil computations, or calculator results by . . . finding "how many" or "how much" to 50.

Allowing students to use calculators to compute sums to 50 undermines the development of arithmetic in these standards.

The development of area in the elementary grade standards is weak. Estimation replaces the logical development of area from rectangle to triangle and then to other polygons. Students are not expected to know how to compute the area of a triangle until sixth grade. In earlier grades, students only estimate areas of polygons other than rectangles. The exact area of a circle is introduced only in the eighth grade. Earlier grade standards call only for estimates of areas of circles. The arithmetic of rational numbers is not addressed until middle school.

Poorly Developed Standards

There is too much emphasis on the use of manipulatives in the upper grades. Seventh-graders are asked to use place value blocks to identify place values for integers and decimals. Use of "models," which we take to mean manipulatives, is *required* as late as ninth grade in order for students to "demonstrate conceptual understanding of mathematical operations . . . on real numbers." Mathematics owes its power and breadth of utility to abstraction. The overuse of manipulatives works against sound mathematical content and instruction.

Seventh-grade students are expected to multiply and divide decimals, but the concept of multiplication and division of fractions is not introduced until eighth grade. The possibility then exists that seventh-graders will utilize rote procedures without understanding the meaning of multiplication or division of decimals.

Another example of poor development in the Alaska standards is a sequence of standards involving measures of angles. Sixth-graders are expected to draw or "measure quadrilaterals" with given dimensions or angles, but they are not expected to measure the degrees of an angle until grade 7.

The upper-grade-level algebra and geometry standards are thin and some of the writing is so poor that meaning is obscured, as in these tenth-grade standards:

> *The student demonstrates conceptual understanding of functions, patterns, or sequences, including those represented in real-world situations, by*
>
> - *describing or extending patterns (families of functions: linear, quadratic, absolute value), up to the nth term, represented in tables, sequences, graphs, or in problem situations*
> - *generalizing equations and inequalities (linear, quadratic, absolute value) using a table of ordered pairs or a graph*
> - *using a calculator as a tool when describing, extending, representing, or graphing patterns, linear or quadratic equations L.*

Probability and statistics are overemphasized at all grade levels, particularly in the lower grades before fractions are well developed. Patterns are also overemphasized and the standards devoted to patterns have little connection to mathematics.

Arizona

Reviewed: *Arizona Academic Content Standards*, March 2003. Arizona provides standards for each of the grades K-8 and a single set of standards for the high school grades.

2005 STATE REPORT CARD

Arizona

Clarity: 2.00	C
Content: 2.00	C
Reason: 2.00	C
Negative Qualities: 2.00	C
Weighted Score: 2.00	Final Grade: C
2000 Grade: **B**	
1998 Grade: **B**	

Arizona has the makings of a good start with these relatively new standards, but there are shortcomings in content coverage and logical development that drag down its grade. These standards are divided into five strands: Number Sense and Operations; Data Analysis, Probability, and Discrete Mathematics; Patterns, Algebra, and Functions; Geometry and Measurement;

and Structure and Logic. Each of these strands further subdivides the standards into "Concepts," some of which are unconventional. For example, within the Data Analysis strand is "Concept 4: Discrete Mathematics (Vertex-Edge Graphs)." Under the Algebra and Functions strand is the group of standards devoted to "Concept 4: Analysis of Change." And within the Structure and Logic strand a collection of standards is labeled, "Concept 1: Algorithms and Algorithmic Thinking."

Making Progress . . . Slowly

A commendable feature of Arizona's standards is that several of the "Concept" categories at lower grade levels are left blank. For example, it is refreshing to find no standards listed under "Concept 2: Probability" at the Kindergarten level, which is too early to introduce this topic. Other states could improve their own standards by emulating this feature.

However, in other cases, standards grouped under "Concepts" are repetitive and lack content. For example, the following standards listed under "Concept 4: Discrete Mathematics (Vertex-Edge Graphs)" appear respectively in each of the grades K-2 and 3-5:

> *Grades K-2*
>
> *Color pictures with the least number of colors so that no common edges share the same color (increased complexity throughout grade levels).*
>
> *Grades 3-5*
> *Color maps with the least number of colors so that no common edges share the same color (increased complexity throughout grade levels).*

Devoting class time for six years of school to coloring pictures and maps in this fashion, perhaps in recognition of the "Four Color Theorem," takes valuable time away from more important topics for elementary school students. Similarly, "Concept 4: Analysis of Change" includes repetitive standards from year to year in the lower grades:

> *Grade 1*
>
> *Identify the change in a variable over time (e.g., an object gets taller, colder, heavier, etc.).*
>
> *Make simple predictions based on a variable (e.g., select next stage of plant growth).*
>
> *Grade 2*
>
> *Identify the change in a variable over time (e.g., an object gets taller, colder, heavier).*
>
> *Make simple predictions based on a variable (e.g., a child's height from year to year).*
>
> *Grade 3*
>
> *Identify the change in a variable over time (e.g., an object gets taller, colder, heavier).*
>
> *Make simple predictions based on a variable (e.g., increases in allowance as you get older).*

At the middle and high school levels, the standards listed under "Concept 4: Analysis of Change" are vague and superficial.

> *Grade 7*
>
> *Analyze change in various linear contextual situations.*
>
> *High School*
>
> *Determine the solution to a contextual maximum/minimum problem, given the graphical representation.*

Finding maxima and minima of functions is an important topic in calculus, but the prerequisites to deal with that topic are not developed in the Arizona standards. Arizona would do better by placing more emphasis on algebra and geometry, topics poorly developed in these standards. For example, there is no mention of completing the square of quadratic polynomials, and little attention to proofs in geometry. With the exception of a single standard calling upon students to "identify the sine, cosine, and tangent ratios" of acute angles, trigonometry is missing.

Inconsistent Coverage

Ironically, there is no mention of the standard algorithms of arithmetic under "Concept 1: Algorithms and Algorithmic Thinking," or in the rest of these standards. The elementary grades do, however, call for whole number and decimal computations, and there is no mention of calculators for this or other purposes, a positive feature for the elementary school grades. Students are expected to "state multiplication and division facts through 9s" and similarly to state other number facts.

The development of decimal arithmetic is poorly coordinated with fraction arithmetic. Fifth-graders multiply and divide decimals, but it is not until sixth grade that they perform these operations with fractions. The possibility then exists that fifth-graders will utilize rote procedures without understanding the meaning of multiplication or division of decimals.

The Structure and Logic strand is mixed. Many of these standards are too broad. High school students are to "analyze assertions related to a contextual situation by using principles of logic" and "construct a simple formal or informal deductive proof," which gives teachers little guidance as to what students ought to do. Similarly, eighth-graders are required to "solve a logic problem given the necessary information."

On the other hand, some of the standards in this strand are concrete and valuable. Eighth-graders also "verify the Pythagorean Theorem using an area dissection argument," an excellent requirement. Second graders learn useful vocabulary words from the standards: "Identify the concepts *some*, *every*, and *many* within the context of logical reasoning," and "Identify the concepts *all* and *none* within the context of logical reasoning."

Arkansas

Reviewed: *Arkansas Course of Study: Mathematics*, 2003; *Curriculum Frameworks: Mathematics*, 1998; *Sample Curriculum Models, K-8*, 1998; *Sample Grade Level Benchmarks*, 1-4, 1998; *Sample Grade Level Benchmarks*, 5-8, 1999. The Arkansas *Framework* consists of broad standards for grade bands K-4, 5-8, and 9-12. Grade-by-grade benchmarks and sample curriculum models for grades K-8 supplement the *Framework*. However, no supplementary documents were available for grades 9-12 at the time of this review.

2005 STATE REPORT CARD

Arkansas

Clarity: 1.50	**D**	
Content: 0.67	**F**	
Reason: 0.00	**F**	
Negative Qualities: 0.75	**F**	
Weighted Score: 0.72	Final Grade:	F
2000 Grade: **D**		
1998 Grade: **F**		

Arkansas' grade for math standards is lower in 2005 than in 2000, though the standards themselves remain largely unchanged. That's because the 2005 reviewers placed heavier weight—as they should have—on the standards' coverage of math content and, in that crucial area, Arkansas' standards are especially weak.

Overall, the Natural State's standards are disorganized, with spotty coverage of algebra in the higher grades and an overemphasis on technology and manipulatives. Mental math is mentioned in a few benchmarks, but in each instance is given equal billing with technology. Take, for example, this first-grade benchmark:

> *Students will demonstrate competency with basic addition and subtraction facts (sums to 18) using mental math and technology.*

Demonstrating "competency with basic addition and subtraction facts" using "technology" works against memorization of the basic facts. The Arkansas *Framework* conveys boundless trust in the power of technology—even in Kindergarten, where this standard appears:

> *Students will use the tools of technology to experience gathering, organizing, and presenting information.*

In sixth grade, "Students will identify, with and without technology, pi (π) as an irrational number. . . ." Essentially this same standard reappears in eighth grade as well, but with reference to $\sqrt{2}$ instead of pi. Standards such as these point to the use of technology as an end in itself, regardless of mathematical necessity or merit. This latter standard in particular undermines mathematical reasoning, since technology cannot establish the irrationality of pi or of any other irrational number. Moreover, while a proof that $\sqrt{2}$ is irrational may be accessible to some high school students, the tools needed to demonstrate the irrationality of pi go far beyond K-12 mathematics.

Manipulatives Run Amuck

The focus on manipulatives is excessive throughout. Mathematics owes its power and utility to abstraction; overusing manipulatives works against sound mathematical content and instruction. For example, in the fifth grade,

> *The student will: add and subtract fractions and/or mixed numbers with and without like denominators using manipulatives, . . . use appropriate software technology to demonstrate competence with rational number computations,*

and

> *use manipulatives to represent fractions (i.e., continuous wholes, equivalent fractions, and discrete sets with fraction bars, attribute blocks, fraction strips, etc.) (e.g., 1/2 of a cake and 1/2 of a dozen eggs).*

It is unclear what is meant by "continuous wholes." A seventh-grade benchmark asks students to "find what percent one number is of another with the use of manipulatives and technology." The use of manipulatives continues to the eighth grade, long after it should have been discontinued.

Patterns and statistics, probability, and data analysis are overemphasized at all levels. Algebra and pre-algebra are underemphasized in middle and high school. The treatment of linear functions relies too much on graphing calculators and manipulatives and too little on symbolic notation and mathematical reasoning. The Pythagorean Theorem is mentioned twice and only in eighth grade, once in the context of formulas for volume and surface area, and once in the context of indirect measurements. No standard requires students to find the roots of a quadratic polynomial, except one eighth-grade standard—and, of course, it allows students to use "manipulatives and appropriate technology" to solve the problem. The Arkansas benchmarks introduce the number line simultaneously with coordinate graphs in grade five, with positive integrals and "common fractions" as coordinates. Strangely, and inconsistently, the "transformation . . . of geometric figures on the coordinate plane (negative and positive numbers)" already occurs in the grade 5 benchmarks.

Finally, some of the Arkansas benchmarks are straight-out nonsense, such as this one for fifth-graders:

> *Students will develop and use strategies for finding the length of straight and curved lines and the perimeter of two and three dimensional objects.*

Three-dimensional objects do not have perimeters (we suspect "surface area" is meant) and if finding the lengths of curves is expected of fifth-graders then a method should be identified, since this is generally an operation far beyond the ability of most students that age.

California

Review: *Mathematics Framework for California Public Schools*, *2000 Revised Edition* provides standards for each of the grades K-7 and for the courses and topics: Algebra I; geometry; Algebra II; Trigonometry; Mathematical Analysis; Linear Algebra; Probability and Statistics; Advanced Placement Probability and Statistics; and Calculus. At the time of this writing, the revised edition of the *Framework* was the latest available, but additional revisions are in progress, including new appendices addressing algebra readiness and intervention programs.

California's standards are excellent in every respect. The language is crystal clear, important topics are given priority, and key connections between different skills and tasks are explicitly addressed. Computational skills, problem-solving, and mathematical reasoning are unambiguously supported and integrated throughout the standards. For example, the fifth-grade standards addressing fraction multiplication and division proceed logically and clearly:

Understand the concept of multiplication and division of fractions.

Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

Sample problems follow the latter standard. Procedural skill, conceptual understanding, and problem-solving are all required here. Another illustration is this Measurement and Geometry standard for fifth grade:

2005 STATE REPORT CARD

California

Clarity: 3.83	A
Content: 3.94	A
Reason: 3.83	A
Negative Qualities: 3.92	A
Weighted Score: 3.89	Final Grade: A

2000 Grade: **A**

1998 Grade: **A**

Derive and use the formula for the area of a triangle and of a parallelogram by comparing it with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by pasting and cutting a right triangle on the parallelogram).

Sample problems immediately follow in the *Framework*, and a fourth-grade Measurement and Geometry standard carefully lays the groundwork for the above standard:

Understand and use formulas to solve problems involving perimeters and areas of rectangles and squares. Use those formulas to find the areas of more complex figures by dividing the figures into basic shapes.

Top-Notch

The elementary grade standards require memorization of the basic number facts and facility with the standard algorithms of arithmetic, including the important long division algorithm. Standards calling for facility with the standard algorithms of arithmetic also ask for understanding of why the algorithms "work," as in this fourth-grade Number Sense standard:

Demonstrate an understanding of, and the ability to use, standard algorithms for multiplying a multi-digit number by a two-digit number and for dividing a multi-digit number by a one-digit number; use relationships between them to simplify computations and to check results.

The K-7 standards build the prerequisites for secondary algebra and geometry systematically and coherently. California aims to place students in Algebra I, or an integrated math course, by eighth grade, but the *Framework* acknowledges on page 199 that this ambitious program is not always appropriate:

One purpose of a seventh grade assessment, as described previously, is to determine the extent to which students are mastering prealgebraic concepts and procedures. Another is to identify those students who lack the foundational skills needed to succeed in eighth grade algebra and need further instruction and time to master those skills. This additional instruction may be provided through tutoring, summer school, or an eighth grade prealgebra course leading to algebra in the ninth grade.

California's *Framework* clearly and appropriately addresses the role of technology. Chapter 9, "The Use of Technology," provides clear guidance on calculator and computer usage that other states would do well to emulate. A section entitled "The Use of Calculators" begins,

The Mathematics Content Standards for California Public Schools was prepared with the belief that there is a body of mathematical knowledge—independent of *technology—that every student in Kindergarten through grade twelve ought to know and know well. Indeed, technology is not mentioned in the Mathematics Content Standards until grade six. More important, the STAR assessment program—carefully formulated to be in line with the standards—does not allow the use of calculators all through Kindergarten to grade eleven.*

The *Framework*, however, does encourage the use of calculators in specific, appropriate circumstances:

It should not be assumed that caution on the use of calculators is incompatible with the explicit endorsement of their use when there is a clear reason for such an endorsement. Once students are ready to use calculators to their advantage, calculators can provide a very useful tool not only for solving problems in various contexts but also for broadening students' mathematical horizons. One of the most striking examples of how calculators can be appropriately used to help solve problems is the seventh grade topic of compound interest.

A Few Flaws

The K-7 standards are not without shortcomings. The standards, pitched at an internationally competitive level, place stiff demands on students that exceed those of most states, and the *Framework* does not elaborate sufficiently on how best to help students who fall behind. Probability and statistics are overemphasized, although not as much as with most other states. For example, these sixth-grade standards stray too far in the direction of social science and away from mathematics:

Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.

Identify data that represent sampling errors and explain why the sample (and the display) might be biased.

California's K-7 mathematics standards are demanding enough without the inclusion of such diversions as data collection.

The section, "Grade-Level Considerations, Grade Four: Areas of Emphasis" has an egregious error that should be corrected, along with supporting material in Appendix A to which the passage refers. On page 135, the paragraph labeled "Fractions equal to one" includes this statement:

When the class is working on equivalent fraction problems, the teacher should prompt the students on how to find the equivalent fraction or the missing number in the equivalent fraction. The students find the fraction of one that they can use to multiply or divide by to determine the equivalent fraction.

Fourth-grade students cannot use multiplication and division of fractions to find equivalent fractions because multiplication and division of fractions are not introduced until fifth grade. Moreover, equivalence of fractions is fundamental to the arithmetic of rational numbers. The concept of equivalence of fractions must be firmly established, using only whole number operations, before multiplication and division of fractions can be defined and explained. However, equivalence of fractions is correctly addressed by the third-and fourth-grade standards themselves.

A Model for States

The *Framework* identifies the high school content intended for all students as Algebra I, Geometry, and Algebra II (although it does allow integrated math courses covering the same topics). The content standards for the more advanced courses are listed by topic (rather than as courses) with the intention that those standards may be collected and combined in a variety of different possible ways. As the document explains:

To allow local educational agencies and teachers flexibility in teaching the material, the standards for *grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. . . . Many of the more advanced subjects are not taught in every middle school or high school. Moreover, schools and districts have different ways of combining the subject matter in these various disciplines. For example, many schools combine some trigonometry, mathematical analysis, and linear algebra to form a precalculus course. Some districts prefer offering trigonometry content with Algebra II.*

The Algebra I, Geometry, and Algebra II standards are exemplary. In Algebra I, students "know the quadratic formula and are familiar with its proof by completing the square." Geometry students prove major theorems including the Pythagorean Theorem. The standards for the more advanced courses are demanding, and can prepare motivated students for university studies and scientific careers.

California's *Framework* is not perfect. But it comes as close to perfection as any set of mathematics standards in the country, and should be a valuable model for other states.

Colorado

Reviewed: *Colorado Model Content Standards for Mathematics*, February 7, 2000. Colorado provides broad standards for each of the grade bands K-4, 5-8, and 9-12, and specific grade-level standards for grades K-8. Colorado also provides an Assessment Framework, not reviewed here because it is used solely as "a guide for test construction."

2005 STATE REPORT CARD

Colorado

Clarity: 1.00	D	
Content: 1.67	D	
Reason: 1.00	D	
Negative Qualities: 1.50	D	
Weighted Score: 1.37	Final Grade:	D
2000 Grade:	D	
1998 Grade:	D	

Colorado's grade has not changed since our last review. The document remains vague and confusing, with a plethora of time-wasting activities and odd development of key mathematical skills. For example, the word "demonstrate" appears 122 times in the document, often in ways that are unclear:

Eighth grade students will pictorially demonstrate the meaning of commonly used irrational numbers.

High school students will demonstrate the relationships among subsets of the real number system, including counting, whole, integer, rational, and irrational numbers, to one another.

The elementary grade standards call for memorization of the basic number facts and require students to be able to add, subtract, multiply, and divide whole numbers using pencil and paper—a positive feature. But calculators are inappropriately introduced in first grade, potentially compromising whole number arithmetic.

The reliance on demonstrations with concrete objects to develop understanding of arithmetic is excessive. For example, third-graders are expected to use concrete objects to "demonstrate and verbally explain addition and subtraction of whole numbers with regrouping for up to four-digit numbers." Even standards calling for the use of concrete objects to understand the concept of even and odd numbers are excessive:

Second grade students will, using objects and pictures, represent whole numbers including odds and evens from 0 to 1,000.

Third grade students will, using objects and pictures, represent whole numbers including odds and evens from 0 to 10,000.

Fourth grade students will, using objects and pictures, represent whole numbers including odds and evens from 0 to 1,000,000.

Grasping the concept of even and odd numbers does not require three years of collecting progressively more objects. The time devoted to collecting and displaying objects and pictures is better spent on other activities.

Poor Development of Fraction Arithmetic

Throughout these standards, the development of fractions is problematic. The fraction standards for grades K-4 rely completely on concrete objects. For example, fourth graders, "using concrete materials, demonstrate addition and subtraction of mixed numerals with common denominators of twelve or less."

The concept of equivalent fractions and practice reducing fractions are not addressed at all until grade 5. This oversight has the potential to undermine student understanding of fractions as names of numbers, and the understanding that different fractions can name the same number.

In fifth grade, students add fractions with the same denominator with pencil and paper, but use "concrete materials" for addition of fractions (proper only) with different denominators. Sixth-grade standards call for addition and subtraction of fractions using pencil and paper for the first time, but ask students only to "demonstrate multiplication and division of proper fractions" using "concrete materials." Hand calculations for the four operations of arithmetic with fractions are expected for the first time only in seventh grade.

The late development of fractions undermines this standard:

Fifth grade students will:

- *demonstrate the meaning of ratio in different contexts*
- *use appropriate notation to express ratios, including a/b, a to b, and a:b*

The ratio of A to B is the division of A by B. Ratios cannot be sensibly developed without a clear concept of division, but division of fractions, including whole numbers, is not introduced until sixth grade—using "concrete materials"—and symbolically only in the seventh grade.

Unhelpful Standards

Some of the algebra standards are mathematically incorrect, such as this one requiring fourth-graders to

Find missing elements of a complex repeating pattern (for example, 1,1,2,3,5,__,13,…).

Without a specific rule for a pattern, there are no correct or incorrect answers, and leading students to believe otherwise does them a disservice.

Some of the probability standards are confusing, such as:

Seventh grade students will:

- *demonstrate that the probability of independent compound events is the same as the product of the probabilities of the two simple events.*
- *demonstrate that the sum of all the probabilities of the events in a sample space is equal to one.*

It is unclear how students are to "demonstrate" the definitions of sample space and independent events. Throughout, data collection and analysis, statistics, and probability are overemphasized relative to other topics. The high school standards in particular give too little attention to algebra, geometry, and trigonometry, but call upon students to be familiar with normal distributions and work superficially with lines of best fit.

Connecticut

Reviewed: *Connecticut Framework: K-12 Curricular Goals and Standards—Mathematics* and *Common Core of Learning—Mathematics*, both published in 1998, contain standards for grade bands K-4, 5-8, and 9-12. These documents are supplemented by a compact disc entitled *Goals 2000, Mathematics Curriculum—PreK through Grade 12*, a curriculum development resource produced in 2002. *Goals 2000* includes sample activities intended to complement Connecticut's standards and National Council of Teachers of Mathematics standards.

Connecticut	**2005** STATE REPORT CARD	
Clarity: 0.67	F	
Content: 0.33	F	
Reason: 0.00	F	
Negative Qualities: 1.00	D	
Weighted Score: 0.47	Final Grade:	F
2000 Grade: **D**		
1998 Grade: **D**		

Connecticut's unchanged standards have fallen in this review because of the heightened emphasis on content, where the Constitution State falls abjectly short. These standards are marked by vagueness and ambiguity. For example, the Common Core goals and standards, which are also repeated in the *Framework*, are no more than broad aspirations for all of the grades K-12, as in this example:

> *Students will use mathematical skills and concepts with proficiency and confidence, and appreciate the power and utility of mathematics as a discipline and as a tool for solving problems.*

Laudable, surely, but this is not a standard, strictly speaking. To be fair, the *Framework* does include more specific performance standards, but they mostly serve to highlight Connecticut's constructivist approach to mathematics education:

> *K-4: Students use real-life experiences, physical materials, and technology to construct meanings for whole numbers, commonly used fractions, and decimals.*
>
> *5-8: Students use real-life experiences, physical materials, and technology to construct meanings for whole numbers, commonly used fractions, decimals, and money amounts, and extend these understandings to construct meanings for integers, rational numbers, percents, exponents, roots, absolute value, and scientific notation.*
>
> *9-12: Students use real-life experiences, physical materials, and technology to construct meanings for rational and irrational numbers, including integers, percents, and roots.*

These standards place on students the heavy burden of constructing the meaning of the real number system. Connecticut students are not expected to have automatic recall of basic number facts, nor are they required to master computational algorithms. Indeed, Goals 2000 advocates that:

> *Instructional activities and opportunities need to focus on developing an understanding of mathematics as opposed to the memorization of rules and mechanical application of algorithms. . . . Technology plays an important role in developing number sense. Students should have opportunities to use the calculator as a teaching and exploration tool. Young children can use the constant feature of most calculators to count, forward or backward, or to skip count, forward or backward. . . . At the 5-8 grade level, students continue to need experiences that involve the regular and consistent use of concrete models.*

Ambiguity Abounds

Still, the *Framework* is not completely devoid of arithmetic and computation requirements. In K-4, for example, students "develop proficiency with basic addition, subtraction, multiplication, and division facts through the use of a variety of strategies and contexts," while in grades 5-8, they "develop, use, and explain procedures for performing calculations with whole numbers, decimals, fractions, and integers." A promising start, but in keeping with the amorphous nature of Connecticut's standards, no procedures or strategies are identified.

The ambiguity of these standards works against the careful development of fractions and credible preparation for algebra. The Pythagorean Theorem is mentioned only once, in a convoluted standard for grades 5-8:

> *Describe and use fundamental concepts and properties of, and relationships among, points, lines, planes, angles and shapes, including incidence, parallelism, perpendicularity, congruence, similarity, and the Pythagorean Theorem.*

Quadratic polynomials and the quadratic equation receive no mention in these standards. Finally, the *Goals 2000* sample activities do little to clarify the mathematical content of the standards and are at best suitable as classroom enrichment activities.

Delaware

Reviewed: *Mathematics Curriculum Framework*, 1995, provides content standards for grades K-10 arranged in grade-level clusters: K-3, 4-5, 6-8, and 9-10, and a half-page appendix of recommendations for grades 11 and 12. Each standard is accompanied by a list of "performance indicators." The *Framework* also contains additional material called "learning events" and "vignettes."

2005 STATE REPORT CARD

Delaware

Clarity: 0.83	**F**
Content: 0.67	**F**
Reason: 0.50	**F**
Negative Qualities: 0.00	**F**
Weighted Score: 0.53	Final Grade: **F**
2000 Grade: **C**	
1998 Grade: **C**	

Delaware seems to be in a state of flux as relates to its state math standards. The authors of the 1998 Fordham report evaluated Delaware's 1995 *Framework* along with revisions made to it in 1996. To the best of our knowledge, and after extensive communications with the state and searches on the state's standards website, we have determined that those revisions to the original *Framework* are no longer available or distributed to teachers. Delaware has another related document called "Desk Reference for Teachers" that was discussed in the 2000 Fordham report, but the authors have decided to omit discussion of the desk reference, as it is mostly a teaching strategies guide. Here, we consider only the 1995 *Framework*. We note that Delaware is expected to develop new standards in the near future, which hopefully will clear up the confusion—and raise Delaware's grade, which has dropped significantly without the 1998 revisions.

Dazed and Confused

Because of the sweeping generality of the content standards and the use of grade-level clusters, Delaware's *Framework* says little about what students should know and be able to do at any particular point in their schooling. The content standards themselves are pompous and unwieldy, filled with words that seem to refer to mathematical tasks, but do not. A typical example:

> *Students will develop an understanding of ALGEBRA by solving problems in which there is a need to progress from the concrete to the abstract using*

physical models, equations, and graphs; to generalize number patterns; and to describe, represent, and analyze relationships among variable quantities.

The Performance Indicators are intended to make the standards more specific, but they add little information. For the above algebra standard, the Performance Indicators for grades 9-10 include some that are meaningless ("develop appropriate symbol sense to use algebraic technology") and some that are hopelessly vague ("describe relationships between variable quantities verbally, symbolically, and graphically, including slope as a rate of change"). Another grade 9-10 Performance Indicator reduces much of a year's worth of algebra to 13 words, leaving all details to the imagination:

Solve linear and quadratic algebraic problems using graphs, tables, equations, formulas, and matrices.

Performance Indicators for Standards 5-10 exhibit these same shortcomings. Some are nonsensical, such as, "Identify patterns for explaining the concepts of computation." Some are vague to the point of meaninglessness, as in, "Compute with real numbers," "Construct and describe displays of data," and "Identify geometric patterns and relationships." Others address substantive topics in an extremely condensed form that gives no hint of the specifics or the level of knowledge required: "Apply similarity, congruence, and proportionality." Clear and specific performance indicators are few and far between; one such is the following: "Compute circumference; areas of triangles, parallelograms, trapezoids, and circles; and surface area and volume of cylinders, triangular and rectangular prisms, and pyramids." More often the Performance Indicators leave one searching for further information about what students are expected to know and be able to do.

Not-Quite-Coherent

The Performance Indicators for Standards 1-4 are even worse. These address four laudable goals: they ask students to solve problems, communicate mathematically, reason mathematically, and make mathematical connections. But each begins with a pretentious and vacuous statement. For example:

Students will develop their ability to make MATHEMATICAL CONNECTIONS by solving problems in which there is a need to view mathematics as an integrated whole and to integrate mathematics with other disciplines, while allowing the flexibility to approach problems, from within and outside mathematics, in a variety of ways.

The Performance Indicators that are intended to make this more specific include the following:

- *Make connections linking conceptual and procedural knowledge.*
- *Integrate mathematical problem-solving with other curricular areas.*
- *Use connections among mathematical topics.*

These are all laudable goals, but such vague exhortations are useless as standards.

While the Delaware standards formally address the major topics in K-12 mathematics, they are too vague to ensure adequate instruction. For example, it is important (for several reasons) that students learn to use the standard long division algorithm. The grade 4-5 standards require students to "use algorithms for addition, subtraction, multiplication, and division with understanding" and more specifically to "divide whole numbers using multi-digit divisors." This statement suggests that students use long division, but does not say so. However, these quoted sentences are prefaced with the phrase "while using appropriate technology," so these computations can evidently be done on a calculator. Indeed, in Appendix B ("Recommended Technology"), the *Framework* recommends for grades K-5 that "each classroom be equipped with . . . grade level appropriate calculators (four function, algebraic operations, and/or fraction capabilities)." Moreover, one cannot adequately use standard algorithms until one has memorized the basic number facts. That essential prerequisite is not required, or even mentioned, by Delaware's standards.

Unlike most state standards, Delaware's *Framework* does a poor job of developing place value in the early grades. Delaware has only a single performance indicator on place value, and it is both general and muddled: "Build whole numbers using the concept of place value using base ten."

While there is a standard on reasoning, the corresponding Performance Indicators are merely statements of general virtues (e.g., "draw and then justify conclusions") unrelated to any particular mathematical content. A few Performance Indicators for other standards also allude to reasoning without actually requiring anything specific ("apply geometric properties and relationships to make conjectures"). Yet for specific topics where clear reasoning is important, such as the Pythagorean Theorem or irrational numbers, the standards do not require students to understand the underlying logic.

Appendix A in the *Framework* is a set of recommendations for eleventh and twelfth grade consisting of educational jargon. Sentences such as, "An expanded symbol system extends and refines the student's ability to express quantitative ideas concisely," render these recommendations entirely useless.

District of Columbia

Reviewed: Standards for each of the grades Pre-K to 8, and for Algebra I, Algebra II, Geometry, Pre-Calculus, and Advanced Placement Calculus. The Algebra I course standards are nearly identical to the Massachusetts Algebra I standards. The standards for Pre-K to 8, Geometry, Algebra II, and Pre-Calculus are split into strands: Number and Operation, Patterns, Functions and Algebra, Data Analysis, Statistics and Probability, Geometry and Spatial Sense, and Measurement.

2005 STATE REPORT CARD

District of Columbia

Clarity: 1.67	**D**
Content: 1.33	**D**
Reason: 1.50	**D**
Negative Qualities: 1.00	**D**
Weighted Score: 1.37	Final Grade: **D**

2000 Grade: **B**

1998 Grade: **D**

The District of Columbia curriculum document displays each standard in one of three columns labeled "Performance Standards," "Essential Skills," or "Technology Integration." The document explains that the Performance Standards "relate to issues of assessment that gauge the degree to which content standards have been attained." The Essential Skills "represent the content standards, which specify 'what students should know and be able to do.'" The Technology Integration standards describe "technological tools students should use and understand."

Technology is a centerpiece of the D.C. mathematics curriculum. The emphasis on technology is extreme and exceeds that of any other state mathematics curriculum document reviewed here. In this curriculum, the Pre-Kindergarten student:

- *identifies various technologies;*
- *demonstrates proper care and handling of technology;*
- *demonstrates familiarity with the computer keyboard;*
- *begins to use the mouse and/or keypad;*
- *demonstrates familiarity with basic calculator keys.*

The Kindergarten standards include all of the above, as well as the following:

- *uses a calculator to represent joining and separating of concrete objects;*
- *uses word processor to create number sentence stories.*

This last item is remarkable because it evidently assumes that Kindergarten students know how to read, write, and type. The inflated demands made of Kindergarten students do not end here. As part of the probability strand, the Kindergartner "identifies the likelihood of a given situation," but without the use of fractions, which are introduced in first grade.

First graders must use the calculator to "generate number facts," "generate and verify simple addition and subtraction number sentences," and " demonstrate the commutative property of addition," among other activities. In third grade, the student:

- *uses the Internet to support learning;*
- *uses a calculator to discover multiple ways to make change;*
- *uses a calculator to determine the perimeter of polygons;*
- *uses the calculator to demonstrate the relationship between fractions and decimals.*

This last requirement is an example of the dearth of reasoning in these standards. No third-grade standard addresses the concept of equivalent fractions or calls upon students to understand that a fraction represents a division. Using a calculator to convert between fractions and decimals under such circumstances can only be a rote exercise. Beginning in fourth grade, students use fraction calculators. This undermines the critical need to be able to calculate with fractions by hand.

To their credit, the standards do require whole number and rational number computations, but there is no mention of the standard algorithms, and, more often than not, it is unclear where the use of calculators is permitted and where it is not. The standards also expect mental calculation and memorization of the basic number facts—both positive features. However, the standards for mental calculation are so vague as to be nearly meaningless. For example, according to a second-grade performance standard, the student "computes answers mentally," and a fourth-grade Essential Skill is, "The student applies mental math and estimation strategies." No elaboration or examples are provided with these directives. The few standards that call for mental calculation or paper-and-pencil calculation are overshadowed by technology requirements. In sixth grade, not even one standard explicitly requires students to carry out a computation by hand.

Many of the standards are repetitive from one grade to the next. Worse, in the examples below, weaker demands are made in eighth grade than in sixth or seventh grade:

> *6th Grade: The student calculates the circumference and area of circles.*
>
> *7th Grade: The student identifies line positions and relationships and the parts of a circle.*
>
> *8th Grade: The student identifies a radius and diameter of a circle.*

How is the sixth-grade student to calculate the circumference and area of a circle without understanding radius and diameter, which are introduced two years later? The seventh-grade standard is far from clear.

Smart Move

The adoption of the Massachusetts Algebra I standards by the District of Columbia is a step in the right direction, and we would encourage the new superintendent of the D.C. public schools, who has proposed replacing the District's standards with those of Massachusetts or California, to replace the entire set.

D.C.'s present Algebra I standards have commendable strengths. However, a weakness both here and in the Algebra II standards is that there is no clear expectation that students will add, subtract, multiply, and divide rational functions, and these are important skills for calculus and beyond. The second listed Algebra I standard was miscopied from the Massachusetts version. It includes an incorrect equation, $413 - 51+6=14$; the correct equation in the Massachusetts version is: $4|3 - 5| + 6 = 14$.

The algebra standards are better than the geometry standards, which lack much of what appears in a traditional Euclidean geometry course. Among the many vague, inappropriate, or poorly conceived geometry standards are these:

> *The Student:*
>
> - *routinely uses tools, software, and online resources to gather, evaluate, analyze, organize, and convey information pertinent to academic and personal interests;*
> - *uses number systems to identify the results of an algorithm;*
> - *describes and constructs repetitive and/or centered patterns and designs;*
> - *solves problems involving enumeration;*

- *creates a database to classify a set of figures in terms of congruence and similarity*

A rare example of a credible geometry standard is:

> *The student uses the Pythagorean Theorem in many types of situations, and works through more than one proof of this theorem.*

But this is a Performance Standard that is supposed to "gauge the degree to which content standards have been attained." Yet, no content standard calls for a proof of the Pythagorean Theorem. Another mismatch between the Performance Standards and the Essential Skills standards (i.e., the content standards) occurs among the fifth-grade standards. A fifth-grade Performance Standard is:

> *The student accurately adds, subtracts, multiplies rational numbers with and without calculators.*

Taken in isolation, this standard is quite reasonable. The problem is that this "assessment standard" is not supported by the fifth-grade content standards, since no fifth-grade content standard asks students to multiply fractions. That topic first appears in sixth grade. Examples such as these detract from the clarity and testability of the District of Columbia standards.

Florida

Reviewed: *Florida Course Descriptions Grades 6-8 and 6-12*, 1997; *Grade Level Expectations for the Sunshine State Standards*, June 1999. Florida provides grade-level standards for each of the grades K-8, standards for the band of grades 9-12, course descriptions for nine different mathematics courses for grades 6-8, and 49 different course descriptions at the high school and adult levels.

Though Florida's standards have not changed since our last review of them, the Sunshine State's grade has slipped partly as a result of the reviewers' heavier weighting of content coverage. Occasional strong coverage of some topics in Florida's statewide standards cannot overcome glaring deficiencies in the whole, an overemphasis on calculators and technology, and a few inexplicable hang-ups that seem disconnected from the main body of mathematical study.

2005 STATE REPORT CARD

Florida

Clarity: 1.33	**D**
Content: 0.67	**F**
Reason: 1.50	**D**
Negative Qualities: 0.50	**F**
Weighted Score: 0.93	Final Grade: **F**

2000 Grade: **D**

1998 Grade: **D**

The elementary grade standards have several positive features. The number line is introduced in the early grades and there is a strong emphasis on place value, including exposure to bases other than 10. Second-graders are expected to memorize single-digit addition facts, the corresponding subtraction facts, and to understand the relationship between addition and subtraction. In third and fourth grade, the multiplication facts are nicely developed, and it is expected that the fourth-grade student "recalls (from memory) basic multiplication facts and related division facts."

However, the elementary grade standards also suffer from serious deficiencies. There is no mention of the standard arithmetic algorithms for whole numbers. The treatment of fractions, while strong in some respects, has serious gaps. Take this fourth-grade standard:

> *The student reads, writes, and identifies fractions and mixed numbers with denominators including 2, 3, 4, 5, 6, 8, 10, 12, 20, 25, 100, and 1000.*

Why the omission of the denominators 7, 9, 11, and other whole numbers? In fifth grade, the student "explains and demonstrates the multiplication of common fractions using concrete materials, drawings, story problems, symbols, and algorithms." It is unclear what the restriction to "common fractions" means. Can Florida fifth-graders be expected to find the product of 1/7 and 1/11, or are these fractions too "uncommon" to warrant attention? Further, the attention paid to division of fractions is marginal. The only fifth-grade standard addressing this topic is:

> *The fifth grade student explains and demonstrates the inverse nature of multiplication and division, with particular attention to multiplication by a fraction (for example, multiplying by 1/4 yields the same result as dividing by 4).*

In sixth grade, the only standard addressing division of fractions is:

> *The sixth grade student knows, and uses models or pictures to show, the effects of the four basic operations on whole numbers, fractions, mixed numbers, and decimals.*

No explanation is given for what is meant by "the effects of the four basic operations." And generally, the Florida standards give no indication that students are expected to achieve fluency in basic calculations involving fractions.

Calculators and Patterns

The unrelenting insistence on use of calculators and computers in the early grades is potentially damaging. The Florida standards expect that the first-grader "uses a calculator to explore addition, subtraction, and skip counting," "uses a calculator to explore number patterns," and "explores computer graphing software." The requirement for calculator use increases in second grade, where among other requirements, the student

> *chooses and explains the computing method that is more appropriate (that is faster, more accurate, easier) for varied real-world tasks (for example, recall of basic facts is faster than using a calculator whereas recording data from survey results may be easier with a calculator).*

Allowing second-graders to choose calculators over paper and pencil work is ill-advised, as the heavy use of calculators in the early grades undermines number sense and arithmetic.

Throughout Florida's standards, the study of patterns is overemphasized, apparently as an end in itself, with little connection to mathematics. Among the standards addressing patterns in second grade alone are:

> *The second-grade student:*
>
> - *recognizes that patterning results from repeating an operation, using a transformation, or making some other change to an attribute.*
> - *predicts, extends, and creates patterns that are concrete, pictorial, or numerical.*
> - *combines two attributes in creating a pattern (for example, size and color).*
> - *transfers patterns from one medium to another (for example, pictorial to symbolic).*
> - *uses a calculator to explore and solve number patterns.*
> - *identifies patterns in the real-world (for example, repeating, rotational, tessellating, and patchwork).*
> - *identifies and generates patterns in a list of related number pairs based on real-life situations (for example, T-chart with number of tricycles to number of wheels). . . .*
> - *explains generalizations of patterns and relationships.*

Not only do the standards dealing with patterns waste precious instructional time, but in some cases they also lead to false understandings, as in this standard for sixth grade:

> *The student . . . given initial terms in a pattern, supplies a specific missing term in the pattern (for example, given first four terms, supplies sixth term).*

Given only the first four terms of a pattern, there are infinitely many systematic, and even polynomial, ways to continue the pattern, and there is no possible incorrect sixth term.

The emphasis on statistics in all grades is excessive, even in Kindergarten, where according to the Florida standards, the student "knows if a given event is more likely, equally likely, or less likely to occur (for example, chicken nuggets or pizza for lunch in the cafeteria)."

Probability should not be introduced until after students have solid foundations in fractions, as probabilities of events are numbers between zero and one.

Other Problems

The development of irrational numbers in middle school is poor and misleading. In seventh grade, the student "describes the meanings of rational and irrational numbers using physical or graphical displays," and "constructs models to represent rational numbers." There are similar eighth-grade standards. Using physical and graphical displays to describe the meaning of irrational numbers is dubious at best, and questionable even for rational numbers at the middle school level. There is no credible development of irrational numbers in these standards.

Of the forty-nine high school and adult course outlines, we examined those for Algebra I, Algebra II, and the honors versions of that course, Honors Geometry, Analytic Geometry, and Trigonometry (regular and the International Baccalaureate versions). These course outlines are little more than a hodge-podge of topics thrown together without cohesion; they are highly redundant from one course to the next. In the Algebra I and II courses, there is no mention of rational functions, or completing the square of quadratic polynomials. Yet students are expected in both Algebra I and II to "understand . . . the basic concepts of limits and infinity," whatever that might mean.

Georgia

Reviewed: *Quality Core Curriculum in Mathematics*, August 26, 2004; *Quality Core Curriculum: Mathematics*, 1998, grades 9-12. In July 2004, the Georgia State Board of Education approved new mathematics standards for each of the grades K-8. The State Board did not approve, at that time, new course standards for grades 9-12, though they exist in draft form. We consider here the new K-8 math standards along with the 1998 standards for high school. For the high school grades, we evaluated the standards for Algebra I, Algebra II, Geometry, and Advanced Algebra and Trigonometry.

2005 STATE REPORT CARD

Georgia

Clarity: 3.33	**A**
Content: 2.67	**B**
Reason: 2.00	**C**
Negative Qualities: 2.00	**C**
Weighted Score: 2.53	Final Grade: **B**

2000 Grade: **B**

1998 Grade: **B**

Georgia's new standards are on the right track, and with further improvements they could be ranked in the top category.

The K-8 standards are clearly organized, concise, and generally well written, although there are rare exceptions, such as this third-grade standard:

> *Understand the concept of perimeter as being the boundary of a simple geometric figure.*

Perimeter is a quantity with units of length, not a boundary.

Georgia's otherwise commendable K-8 standards are marred by directives for calculator use, unspecified technology, and requirements to use manipulatives in all grades. For example, at each grade level:

> *Students will create and use pictures, manipulatives, models and symbols to organize, record, and communicate mathematical ideas.*

Ultimately, the goal of elementary school mathematics is for students to manipulate numbers, not objects, in order to solve problems. That is what they will need to do when they leave school. This is even more the case for middle school math. Georgia's excellent middle school standards are seriously undermined by this requirement, especially with regard to algebra instruction, the main focus of the eighth grade standards. The standards for the middle grades should insist on the use of mathematical symbols and equations, not manipulatives. Including manipulatives at this level works against sound instruction and the abstract nature of mathematics itself, particularly algebra.

Calculators are explicitly introduced in first grade with a boilerplate standard subsequently repeated for all grades 2-8:

> *Determine the most efficient way to solve a problem (mentally, or with paper/pencil, or calculator).*

If elementary school students are allowed to decide the most efficient way to solve problems, what prevents them from choosing a calculator every time? One vague fourth grade standard does ask for some computational ability without calculator assistance:

> *Students will further develop their understanding of division of whole numbers and divide in problem-solving situations without calculators.*

However, aside from ambiguously worded requirements to memorize the basic number facts, no other standard for grades K-8 specifies what students should be able to do without calculator assistance. This leaves open the possibility that all else can be done with calculators. Further, no mention is made of the important standard algorithms of arithmetic at any grade level.

The development of fractions and decimals is generally good, but uneven. The third-grade standards on this topic are excellent, with an unusually clear development of decimal system notation. However, there are problems with the fifth-grade standards:

> *Students will continue to develop their understanding of the meaning of common fractions and compute with them.*
>
> *a. Understand division of whole numbers can be represented as a fraction* (a/b = a ÷ b).
>
> *b. Understand the value of a fraction is not changed when both its numerator and denominator are multiplied or divided by the same number because it is the same as multiplying or dividing by one.*
>
> *c. Find equivalent fractions and simplify fractions.*
>
> *d. Model the multiplication and division of common fractions.*

How is Part A to be achieved? One argument is straightforward if division of fractions was already developed: $a \div b = a \times 1/b = a/b$. But the only reference to division of fractions in the K-5 standards is Part D above. What are "common fractions" and what does it mean to "model" their multiplication and division? Does Part D mean that multiplication and division of fractions should be carried out only in special cases with manipulatives? Is the intention here to treat division (incorrectly) as repeated subtraction? No standard for fifth grade (or below) asks students to be able to multiply or divide fractions by hand, or to know the "invert and multiply" rule for division. If Part A of the above quoted standard is to be achieved in some other way, then what meaning is given to the expression $a \div b$ at the fifth-grade level? What is the definition of fraction division at this level? This is not made clear.

Part B is also problematic and casts Part C into doubt. The concept of equivalent fractions must be developed before fraction multiplication (which is only "modeled" at the fifth-grade level for "common fractions" rather than defined in general). Using fraction multiplication to explain the concept of equivalent fractions is circular, since fraction multiplication cannot be defined properly until the concept of equivalent fractions is already developed.

The grade 6-8 standards are strong. Algebra and geometry topics are well developed and appropriate for their grade levels. Setting aside the failure of these standards to guide calculator use, the arithmetic of rational numbers is fully developed. Surprisingly there is no mention of irrational numbers, though that topic is taken up in the high school standards.

The high school course standards cover a broad range of topics, including roots of quadratic polynomials, the arithmetic of rational functions, conic sections, trigonometry, complex numbers, logarithms and exponentials, the Fundamental Theorem of Algebra, and a variety of topics in geometry. The writing is sometimes vague, however, as in the second Algebra I standard:

> *Solves problems that link concepts to one another and to practical applications using tools such as scientific or graphing calculators, computers, and manipulatives.*

The standards for the high school courses are weakened by poor-quality sample lesson plans that overemphasize the use of graphing calculators, even for linear functions. The geometry standards calling for proofs are separate from those identifying content, a negative feature. Probability and statistics standards are out of place in the standards for algebra and geometry. We hope these problems will be corrected in the new high school standards being developed at the time of this writing.

Hawaii

Reviewed: *Curriculum Framework for Mathematics*, Draft May 2003; *Grade Level Performance Indicators* (GLPI), Revised Draft March 2004; *Scope and Sequence for Mathematics* (SS), Draft, May 2003; *Standards Toolkit Instructional Guides* (IGs), Draft, May 2003. The most recent of these documents, GLPI, provides Performance Indicators for each of the grades K-12. The Scope and Sequence topics are based on the GLPI Performance Indicators. The IGs provide performance assessment tasks and sample instructional strategies for each Grade Level Performance Indicator.

2005 STATE REPORT CARD

Hawaii

Clarity: 1.00	D
Content: 0.33	F
Reason: 0.00	F
Negative Qualities: 0.50	F
Final Weighted Score: 0.43	Final Grade: F
2000 Grade: C	
1998 Grade: F	

There is little that can be salvaged in Hawaii's mathematics standards.

Hawaii organizes its expectations according to a complicated hierarchy. There are 14 general standards that elaborate 5 content strands: Numbers and Operations; Measurement; Geometry and Spatial Sense; Patterns, Functions, and Algebra; Data Analysis and Probability. Benchmarks for bands of grades, such as grades 4-5 and 6-8, then further refine the 14 standards. Detailed Performance Indicators specific to individual grade levels and high school courses then add further specificity to the Benchmarks. In addition, the Process Standards, Problem Solving, Reasoning and Proof, Communication, Connections, and Representation are intended to be incorporated into the teaching of the content strands.

Sinking, Not Swimming

The *Framework* articulates philosophical perspectives on the teaching of mathematics, generally aligned with constructivist trends of education colleges. On page ix, the *Framework* promises that its "Curriculum content recognizes multicultural, global views as well as the Western/European viewpoint and culture." Under the heading "Beliefs and Assumptions About Learning," the document minimizes the importance of mathematical prerequisites, explaining that:

> *Learning higher-level mathematics concepts and processes are not necessarily dependent upon "prerequisite" knowledge and skills. The traditional notion that students cannot learn concepts from Algebra and above (higher-level course content) if they don't have the basic skill operations of addition, subtraction, etc. has been contradicted by evidence to the contrary.*

No such evidence is cited, but this point of view is consistent with the deficiencies of the lower-grade Performance Indicators and Benchmarks. To start, on page 23, one finds, "Technology is essential in teaching and learning mathematics," which would have surprised Newton.

The *Framework* recommends specific math textbooks and programs evidently aligned to its standards and viewpoints about teaching (page 53). Many of these have been widely criticized by professional mathematicians, such as *Interactive Mathematics Program*, *Connected Mathematics*, *Investigations in Number, Data, and Space*, and other similar controversial programs.

A glossary of mathematical terms appears near the end of the Framework, which is badly in need of correction and improvement. A sample entry is:

> Inclusive events (inclusion): *Two events, A and B, are inclusive if the outcomes of A and B are the same. The probability of two inclusive events, A and B, occurring is found as follows:* P(A or B) = P(A) + P(B) – P(A and B).

The *Framework* does not provide details about K-8 content, but Appendix C has "Suggested High School Course Outlines." While lacking sufficient detail, these outlines are nevertheless far superior to the grade 9-12 Performance Indicators in the newer GLPI, as well as the other documents. Unfortunately, the high school course outlines found in the *Framework* are inconsistent with the GLPI Benchmarks and Performance Indicators. Since this latter document is more recent, our numerical scores for the high school standards are based on the Performance Indicators in the GLPI.

Where's the Content?

The GLPI high school Performance Indicators for Pre-Algebra, Algebra I, and Algebra II are highly repetitive and inappropriate. Emphasis is given to probability and statistics, vectors and matrices, and error analysis, but almost no attention is given to high school algebra. The Geometry course emphasizes matrices and vectors, but has few standard topics in geometry. The following peculiar Performance Indicator appears for each of the courses Pre-Algebra, Algebra I, Algebra II, Trigonometry, and Analytic Geometry:

> *The student uses the concept of infinity in a number of ways (e.g. unbounded behavior or function, sequences, as a limit of a variable).*

Given the near absence of the development of elementary algebra in the algebra courses, this standard is out of place and wildly inflated.

The middle school grade Performance Indicators are also highly repetitive. For example, the two indicators, "organizes collections of data" and "chooses, creates, and uses various representations of data" are listed for each of the grades 6, 7, and 8. Too much attention is given to estimation and not enough to exact calculations. Scant attention is given to the arithmetic of rational numbers. The two Indicators in sixth grade for this critical topic are:

> *The Student:*
>
> *Describes situations when addition, subtraction, multiplication, and division involving rationals are appropriate.*
>
> *Selects and uses appropriate strategies for computing with rationals.*

What are the appropriate strategies for computation? Is pressing calculator buttons an appropriate strategy? What does it mean to "describe situations" in the first indicator above? Constructivist dogma is taken to an extreme in the sixth-grade indicator, "The Student discovers the definition and description of fundamental shapes." Definitions cannot be discovered; they must be provided as a foundation for further learning.

In seventh grade, the student "experimentally determines the formula [*sic*] for circumference and area of a circle." This directive is followed later by a high school indicator that asks students to determine formulas for the volumes of spheres, cylinders, and cones experimentally. Experiments cannot supplant the mathematical reasoning required to deduce such formulas. The few requirements in these standards that do call for mathematical reasoning are so vague and poorly formulated that it is difficult to know what is intended, as in the eighth-grade requirement that the student "describes and applies geometric ideas and relationships to solve problems (e.g., polygons, similarity, Pythagorean Theorem, proof)." What proof? Does this indicator ask for a proof of the Pythagorean Theorem, for example? One can only guess.

Calculators are introduced in second and third grade, when students are called upon to "develop and use strategies, including mental arithmetic and calculator, and invent algorithms to find sums and differences up to one hundred." No mention is made of the important standard algorithms of arithmetic. Instead, students invent their own methods throughout the grades. A bright spot in these mostly dismal requirements is that fourth-graders are expected to memorize the multiplication and division facts, but the standards are ambiguous about the addition facts with which students need only show "facility," rather than demonstrate recall.

Hawaii follows an unfortunate trend among states of introducing calculus concepts too early and without necessary prerequisites. Fourth grade students are asked to identify and describe "situations with varying rates of change such as time and distance." It makes no sense to teach calculus concepts when much of arithmetic and algebra is ignored.

Idaho

Reviewed: Idaho's 2003 mathematics standards appear in several documents available from Idaho's Department of Education website. Among these is *K-12 Achievement Standards: Teacher's Guide to Math*, which includes "Samples of Applications," or examples of teaching strategies. Standards are provided for each of the grades K-8 along with a single set of standards for grades 9-12.

2005 STATE REPORT CARD

Idaho

Clarity: 1.67	D	
Content: 0.67	F	
Reason: 1.00	D	
Negative Qualities: 1.50	D	
Weighted Score: 1.10	Final Grade:	D
2000 Grade: -		
1998 Grade: F		

Idaho's subpar standards begin on an unfortunate note with the definition of "Appropriate Technology":

> *May include paper and pencil, graph paper, simple calculators, graphing calculators, computers with spreadsheets, or even specialized mathematics software such as Geometer's Sketchpad or Maple. It is the decision of school districts and teachers to determine which tools are most appropriate for both instruction and application.*

A standards document should play a stronger role in defining what is and what is not appropriate technology.

The Wrong Priorities

Calculators play a central and overwhelmingly negative role in Idaho's standards. In Kindergarten through second grade, students use a four-function calculator. A third-grade standard, repeated for subsequent grades, under the heading "Perform computations accurately," says: "Select and use an appropriate method of computation from mental math, paper and pencil, calculator, or a combination of the three." Beyond the commendable requirement to memorize the basic number facts, computational fluency without use of a calculator is not explicitly required by any of these standards. For one fourth-grade standard, students "Use a computer application to chart or graph the different colors of M&Ms found in a bag." Essentially this same activity—sorting M&Ms by color—is also offered for eighth-graders in another standard. In fifth grade, students can "use a calculator to explore the pattern when multiplying with multiples of 10, for instance *400 x 20=8,000*."

The first-and second-grade standards prematurely introduce estimation and "reasonableness" of results. These skills are more appropriately developed, together with the concept of rounding, in higher grades, after students have had experience making exact calculations by hand. In the elaboration of one first-grade standard, the example is provided: "Given 9-4, would 10 be a reasonable number?" Similarly, in second grade one finds: "Given subtraction problem, 38-6, would 44 be a reasonable answer?" These examples are misguided. For these subtractions, the correct answer is the only reasonable answer. The notion of "reasonableness" might be addressed in grades one and two in connection with measurement, but not in connection with arithmetic of small whole numbers.

Probability and statistics standards are overemphasized throughout, with probability standards in the lower grades particularly misplaced. In Kindergarten, students are already expected to:

> *Understand basic concepts of probability.*
>
> *a. Predict and perform results of simple probability experiments.*

Probabilities are numbers between zero and one. It makes no sense to teach probability to students who have not been exposed to fractions. In fact, fractions are not introduced until fourth grade, when students

> *use concrete materials to recognize and represent commonly used fractions.*

Similar probability standards are given for first and second grade, and a misleading activity is suggested for second grade:

> *Use 6 coins to record heads or tails. After 9 trials, predict the tenth outcome.*

The intention is unclear here. How are students to predict the outcome of the tenth independent trial of this experiment?

Mediocre Math for the Middle Years

Little progress is made in middle school. The standards are bogged down and repetitive. For example, it is suggested that students play the game "Battleship" in both fifth and sixth grade in order to learn how to plot points in the coordinate plane. In both sixth and seventh grade, students "explore the use of exponents." In sixth grade, a suggested activity is "Express 5^2 as factors of 5 and in standard form," while in seventh grade, the suggested activity is "Express 5^3 as factors of 5 and in standard form." The extra factor of five in seventh grade represents no progress at all over sixth grade. The frequent directive to "explore" these notions is also not testable. It is only in eighth grade that students are finally expected to "understand and use exponents." In all three of the grades 6, 7, and 8, students "apply dimensional analysis," with nearly identical activities suggested for each grade.

In the algebra strand, variables are introduced in a proper way in fifth grade, in sentences containing a single unknown. Not much happens beyond that in the algebra strand for grades 5-8. In grade 8, the examples are still very simple, as in the following: "Evaluate an expression such as $2x+y$ when given values for x and y; simplify expressions such as $3a+4b-5a+6b-7$; solve equations such as $12x-5=31$." In some cases, the examples are nonsensical:

> *Understand and use variables in expressions, equations, and inequalities.*
>
> *Sample of Application: . . . If B represents the number of boys in the class, and G represents the number of girls in the class, write an equation and solve it in the number of students in the classroom.*

Technology is overemphasized in middle school, as in the elementary grades. For example, eighth graders

> *explore graphical representation to show simple linear equations. . . .*
>
> *Sample of Application: i. Use technology to create a graph of linear relations.*

Students, of course, should be able to graph linear equations by hand—a skill crucial to the process of understanding what the graph of an equation represents.

Finally, in the single set of standards for the high school grades 9-12, the quality of the document deteriorates precipitously. The critical subjects of algebra and geometry receive scant attention. The algebra of polynomials is only weakly developed, and geometric proof is missing entirely. Diversionary topics and empty rhetoric appear in place of solid content. In one standard, under "Apply appropriate technology," we find "Use computers for manufacturing process control." In the measurement strand, students are asked to "build and use scale models." In another example, students "use linear programming to find feasible regions for manufacturing processes." And another standard is: "Use appropriate technology to employ simulation techniques, curve fitting, correlation, and graphical models to make predictions or decisions based on data." These vague directives are no substitute for solid coverage of crucial concepts and operations.

Illinois

Reviewed: *Illinois Learning Standards*, July 25, 1997; *Performance Descriptors*, 2002. The Illinois Learning

Standards list expectations for five categories of students: early elementary, late elementary, middle/junior high school, early high school, late high school. These categories do not correspond to specific grade spans. We did not review the *Illinois Assessment Framework* since it is a guide to test creation, not a set of standards, strictly speaking.

Illinois	**2005** STATE REPORT CARD	
Clarity: 1.50	**D**	
Content: 2.00	**C**	
Reason: 1.00	**D**	
Negative Qualities: 2.50	**B**	
Weighted Score: 1.80	Final Grade:	C
2000 Grade: **D**		
1998 Grade: **D**		

The element added to Illinois' standards since our last review—the 2002 *Performance Descriptors*—does add some specificity to the generally poor *Learning Standards* and thus helps to improve Illinois' grade. Unfortunately, it also adds confusion to Illinois' standards: When are students supposed to learn what? The following prefatory material from the *Performance Descriptors* describes age categories for the descriptors:

> *Performance Descriptors identify ten developmental stages for each Learning Standard: stages A-H correspond to grades 1-8 and stages I and J correspond to early and late high school. We used stages instead of grade levels to accommodate the range of development that exists in every classroom. For example, we would recommend that a 3rd grade teacher begin by looking at Stage C, which was written with third graders in mind. But we would also recommend looking at Stages B and D.*

The standards associate bands of grade levels to each of the stages A-H. It is commendable for teachers to be aware of preceding and subsequent grade-level requirements, but not at the expense of clear-cut grade-level expectations for students.

Content Deficiencies

The standards, taken alone, are terse and frequently indefinite, as illustrated by the early elementary standard, "Select and perform computational procedures to solve problems with whole numbers." In the lower grades, there are serious deficiencies in the treatment of arithmetic; for example, students are not expected to memorize the basic number facts. Calculator use is promoted beginning in the earliest grades, as seen in this "Stage A" standard, which corresponds to grades one and two: "Utilize a calculator for counting patterns." Then, implausibly, an early elementary standard calls upon students to "Solve one- and two-step problems with whole numbers using addition, subtraction, multiplication, and division." It is unclear how first-and second-graders could carry out division *without* the use of calculators or similar inappropriate technology. Paper and pencil calculations are also expected, but there is no mention of the standard algorithms of arithmetic in either the *Standards* or the *Performance Descriptors*. Instead, the latter document, for example, encourages students to, "Select and use one of various algorithms to add and subtract."

The measurement standards for the elementary school age groups, and more generally all of the grades, are well written and comprehensive. However, probability is introduced prematurely in the early elementary grades, as illustrated by this standard: "Describe the concept of probability in relationship to likelihood and chance." Since probabilities are numbers between 0 and 1, the introduction of probability standards before students have a clear understanding of fractions has no justification.

The middle grade standards and descriptors cover a broad range of topics, including rational number arithmetic, geometry, and pre-algebra, but there is too much reliance on technology.

The standards and descriptors associated with high school are relatively strong. Algebra, geometry, trigonometry, and probability and statistics are covered well. However, mathematical reasoning is weak in these standards. One set of standards and an analogous set of descriptors address mathematical proofs only generically, such as the descriptor, "Develop a formal proof for a given geometric situation on the plane." Unfortunately, such standards and descriptors are set apart from the content topics to which they could most naturally be applied. For example, one Stage I descriptor asks students to "identify and apply properties of medians, altitudes, angle bisectors, perpendicular bisectors, and midlines of a triangle." Apparently students are expected to "identify and apply" theorems related to these topics, without necessarily understanding their proofs.

Indiana

Reviewed: *Indiana Mathematics Academic Standards*, Approved September 2000, subsequently updated. Indiana provides standards for each of the grades K-8, and for each of the secondary courses, Algebra I, Geometry, Algebra II, Integrated Math I, II, and III, Pre-Calculus, Probability and Statistics, and Calculus.

2005 STATE REPORT CARD

Indiana

Clarity: 3.67	A
Content: 3.83	A
Reason: 4.00	A
Negative Qualities: 3.75	A
Weighted Score: 3.82	Final Grade: A
2000 Grade: C	
1998 Grade: C	

Indiana's 2000 revision of its standards was a remarkable success, vaulting it from the middle rank of states to near the top of the pack. These standards have many admirable features. The writing is generally clear and the content is excellent and well organized. Mathematical reasoning is implicitly or explicitly required in many of the content standards. One particularly commendable feature, that other states would do well to emulate, is that the standards for grades K-3 do *not* have a probability and statistics strand.

The elementary grade standards require mastery of the basic number facts, mental calculation, and skill with the standard algorithms for addition and subtraction. Facility with the standard algorithms for multiplication and division is required in the case of single-digit divisors for division and a single-digit factor for multiplication as indicated by these fourth-grade standards:

> *Use a standard algorithm to multiply numbers up to 100 by numbers up to 10, using relevant properties of the number system. Example:* 67 x 3 = ?

> *Use a standard algorithm to divide numbers up to 100 by numbers up to 10 without remainders, using relevant properties of the number system. Example:* 69 ÷3 = ?

A shortcoming is that the fifth-grade standard for multiplication and division of whole numbers in general leaves students free to choose their own methods:

> *Solve problems involving multiplication and division of any whole numbers. Example:* 2,867 ÷ 34 = ? *Explain your method.*

The absence of any requirement to learn the long division algorithm for whole numbers in general slightly undermines the foundations for the understanding of irrational numbers in later grades. Computations with decimals are required in sixth grade, but with no specified methods:

> *Multiply and divide decimals. Example:* 3.265 x 0.96 = ?, 56.79 ÷ 2.4 = ?

The development of fractions is fast-paced. By third grade students are expected to:

> *Show equivalent fractions using equal parts. Example: Draw pictures to show that* $^3/_5$, $^6/_{10}$, *and* $^9/_{10}$ *are equivalent fractions.*

This may even be overly ambitious, since memorization of all of the multiplication facts is not expected until the following year in fourth grade. Third-graders also add and subtract fractions with the same denominator.

Fifth-graders multiply and divide fractions and add and subtract mixed numbers and decimals. An example of the commendable attention given to reasoning by the Indiana standards is illustrated in this fifth-grade standard:

Use models to show an understanding of multiplication and division of fractions. Example: Draw a rectangle 5 squares wide and 3 squares high. Shade $^4/_5$ of the rectangle, starting from the left. Shade $^2/_3$ of the rectangle, starting from the top. Look at the fraction of the squares that you have double-shaded and use that to show how to multiply $^4/_5$ by $^2/_3$.

Minor Complaints

The treatment of areas in the lower grades is one of the few defects of the lower elementary grade standards. A legitimate (though redundant) fourth grade standard is, "Know and use formulas for finding the areas of rectangles and squares," but in grades 3 and 2 respectively, one finds:

Estimate or find the area of shapes by covering them with squares. Example: How many square tiles do we need to cover this desk?

Estimate area and use a given object to measure the area of other objects.

Example: Make a class estimate of the number of sheets of notebook paper that would be needed to cover the classroom door. Then use measurements to compute the area of the door.

The concept of area should be developed more carefully than indicated in this last example especially. Sheets of notebook paper are not square and the area of the door, calculated by multiplying length times width, is not the number of notebook sheets needed to cover it. Area should be introduced initially for rectangles with positive whole number sides and then determined exactly. Only after that should students be expected to estimate areas, especially when the exact area is not a whole number of square units.

The middle school grade standards and secondary course standards are for the most part well crafted and complete. However, examples that accompany them leave room for improvement, as illustrated in these two consecutive Algebra I standards:

Understand the concept of a function, decide if a given relation is a function, and link equations to functions. Example: Use either paper or a spreadsheet to generate a list of values for x and y in $y = x^2$*. Based on your data, make a conjecture about whether or not this relation is a function. Explain your reasoning.*

Find the domain and range of a relation. Example: Based on the list of values from the last example, what are the domain and range of $y = x^2$?

Spreadsheets have no legitimate role to play in deciding whether $y = x^2$ is a function and what its natural domain and range are.

A Plethora of Probability

The Data Analysis and Probability strand that runs from fourth grade to eighth grade, while better than analogous strands for many other states, is nevertheless overblown. For example, in eighth grade, students are expected to:

Represent two-variable data with a scatterplot on the coordinate plane and describe how the data points are distributed. If the pattern appears to be linear, draw a line that appears to best fit the data and write the equation of that line.

To develop the topic of lines of best fit properly is college-level mathematics, and to do it in other ways is not mathematics. Moreover, some of the data analysis standards stray too far from mathematics in the direction of social science, such as this eighth-grade standard:

Identify claims based on statistical data and, in simple cases, evaluate the reasonableness of the claims. Design a study to investigate the claim. Example: A study shows that teenagers who use a certain brand of toothpaste have fewer cavities than those using other brands. Describe how you can test this claim in your school.

A few of the standards are poorly stated, such as this eighth-grade example:

Understand that computations with an irrational number and a rational number (other than zero) produce an irrational number. Example: Tell whether the product of 7 and π *is rational or irrational. Explain how you know that your answer is correct.*

or this standard for Integrated Math:

Know and use the relationship $\sin^2 x + \cos^2 x = 1$. *Example: Show that, in a right triangle,* $\sin^2 x + \cos^2 x = 1$ *is an example of the Pythagorean Theorem.*

In the above standard, the phrase "in a right triangle" is out of place. In a similar vein, the glossary needs careful editing (e.g., "prime number" and "composite number" are not correctly defined).

Despite these minor flaws, Indiana's excellent mathematics standards are among the best in the nation.

Kansas

Reviewed: *The Kansas Curricular Standards for Mathematics*, revised July 2003. This document contains detailed standards for each of the grades K-8, and a single set of standards for the combined grades 9 and 10. The document also includes guidelines "to address a wide variety of response and communication modalities or methods used by students who qualify for the alternate assessment."

2005 STATE REPORT CARD

Kansas

Clarity: 1.67	D
Content: 0.94	F
Reason: 0.33	F
Negative Qualities: 0.25	F
Weighted Score: 0.83	Final Grade: F
2000 Grade: **A**	
1998 Grade: **D**	

The Kansas standards for math sprawl across 318 pages, the result of a recent revision. Alas, the Sunflower State would have been better off keeping its old standards, which earned top marks from our reviewers in 2000. The new ones are an organizational disaster.

The distinction between "Knowledge Base Indicators" and "Application Indicators" is artificial and unhelpful, and despite their great length these standards give almost no attention to mathematical reasoning. As an example, students are expected to use the Pythagorean Theorem and the quadratic formula, with no guidance as to how those results may be deduced or proven.

Technology is grossly overemphasized at all grade levels. The "Vision Statement" in the introduction to the *Framework* makes clear that "technology will be a fundamental part of mathematics teaching and learning." Undue attention is also given to tesselations in each of the grades 7 through 10. Patterns, probability and statistics, and physical models are overemphasized at all grade levels. Multiplication and division of fractions is not expected until sixth grade, but sample probability problems in the lower grades require multiplication of fractions for solution. Students are not explicitly called upon to memorize the single-digit arithmetic facts, or to use the standard arithmetic algorithms.

Models or Manipulatives?

The phrase "Mathematical Models" appears in the document 572 times. A benchmark repeated several times is, "Models—The student develops and uses mathematical models including the use of concrete objects to represent and show mathematical relationships in a variety of situations." It should be noted that the scientific use of "mathematical models" and "mathematical modeling" has nothing to do with the manipulatives ("concrete objects") referenced here.

The vast array of physical devices that students are required to manage in order to "show mathematical relationships in a variety of situations" includes place value mats, hundred charts, base ten blocks, unifix cubes, fraction strips, pattern blocks, geoboards, dot paper, tangrams, and attribute blocks. By tenth grade, students must use "process models (concrete objects, pictures, diagrams, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate grids) to model computational procedures, algebraic relationships, and mathematical relationships and to solve equations."

The requirements to use so many manipulatives is both daunting and of negligible educational value. In the lower grades, use of physical objects in moderation can illuminate mathematical principles, but as students progress they should move beyond manipulatives. It is unclear from reading this document whether the point of manipulatives is to illuminate mathematical principles, or whether mathematical principles serve mainly as prerequisites to using manipulatives.

2005 STATE REPORT CARD

Kentucky

Clarity: 1.83	**C**
Content: 2.33	**C**
Reason: 1.00	**D**
Negative Qualities: 1.50	**D**
Weighted Score: 1.80	Final Grade: C

2000 Grade: **B**

1998 Grade: **D**

Kentucky

Reviewed: *Learning Goals and Academic Expectations; Core Content for Assessment*, 1999; *Program of Studies Mathematics*, updated June 22, 2004; *Combined Curriculum Document*, updated June 29, 2004. The Learning Goals are overarching themes that apply to all grade levels. Each of the six *Learning Goals* is supported by more detailed statements called "Academic Expectations." The *Program of Studies* outlines minimum high school graduation requirements. *Core Content for Assessment* presents the essential content standards and is the basis for the Kentucky mathematics assessment. The *Combined Curriculum Document* was created to reduce the difficulties teachers and parents face in attempting to understand what the grade-level expectations are from these three different documents. Taken together, these documents provide standards for the band of grades K-3, each of the grades 4-8, Algebra I, Geometry, Algebra II, and some standards for eleventh grade.

Kentucky's numerous documents—all of which refer back to each other in confusing ways—are generally mediocre. As one example of the confusion, the grade-level standards in the curriculum documents make direct reference to the Academic Expectations. But the Academic Expectations themselves are broad and vague. Typical examples are:

> *Students understand number concepts and use numbers appropriately and accurately.*
>
> *Students understand space and dimensionality concepts and use them appropriately and accurately.*
>
> *Students understand measurement concepts and use measurement appropriately and accurately.*

Each expectation is followed by lists of elementary, middle, and high school standards and activities designed to promote understanding. For example, an activity for the second Expectation quoted above is:

> *Draw a coordinate system on a parking lot or football field; assign all students a different x value; students stand along the x-axis, and teacher calls out an equation in y-intercept form; students walk to y value to represent the graph.*

Another activity, for the third Expectation quoted in the list above, is:

> *Investigate the average number of kernels on an ear of corn. Compare findings with [sic] number of kernels found in the average serving of canned corn/popped corn.*

These activities, while possibly entertaining, are mathematically shallow and time-consuming—as are many other suggested activities listed in the Academic Expectations.

Spotty Standards

Kentucky's elementary grade standards require memorization of both basic number facts and multi-digit whole number calculations, a positive feature. However, no reference is made in any of the curriculum documents to the standard algorithms of arithmetic, a serious shortcoming. Furthermore, calculators are encouraged in elementary school, when students should be memorizing the basic number facts.

The notion of equivalent fractions is introduced in fourth grade, along with addition and subtraction of fractions with the same denominators, but only through the use of manipulatives and diagrams. Symbolic calculations are expected beginning in fifth grade, but the use of manipulatives is overemphasized in the middle and high school standards, as illustrated by these examples:

> *Students will extend and apply addition, subtraction, multiplication, and division of common fractions and decimals with manipulatives and symbols (e.g., mental, pencil and paper, calculators). (Program of Studies, Grade 6)*

> *Students will perform the following mathematical operations and/or procedures accurately and efficiently, and explain how they work in real-world and mathematical situations: Model equations and inequalities concretely (e.g., algebra tiles or blocks), pictorially (e.g., graphs, tables), and abstractly (e.g., equations).(Core Content for Assessment, Grade 8)*

> *Students will solve one-variable equations using manipulatives, symbols, procedures, and graphing. (Program of Studies, Algebra I)*

The use of verbs like "explore" and "investigate" renders many of Kentucky's standards so vague as to be meaningless, such as this sixth-grade standard from *Program of Studies*: "Students will explore the concept of variable, expression, and equation." However, many of Kentucky's standards, particularly for the middle grades, demand specific and appropriate knowledge and competencies, such as this eighth-grade standard from the *Core Content for Assessment*: "Students will describe properties of, define, give examples of, and apply real numbers to both real-world and mathematical situations, and understand that irrational numbers cannot be represented by terminating or repeating decimals."

In high school, the Algebra I course standards give too much attention to curve-fitting and statistics, and too little to fundamental topics like finding the roots of quadratic polynomials and the arithmetic of rational functions. The Algebra II course standards require factoring of quadratic polynomials and use of the quadratic formula, but completing the square and deriving the quadratic formula are not required. Complex numbers are not mentioned in these standards and calculation of a quotient of polynomials is expected only in the case that the divisor is monomial.

The standards for high school geometry require students to use the Pythagorean Theorem and its converse, but there is little or no mention of proofs of those theorems or other geometry theorems. The Pythagorean Theorem is "discovered" in the eighth grade, however. Trigonometric functions receive scant attention beyond applications to problems involving right triangles.

Louisiana

Reviewed: *Content Standards Foundation Skills*, 1997; *Grade Level Expectations*, 2004. The Louisiana state standards for mathematics are defined by two documents: the *Louisiana Mathematics Framework* dated May 22, 1997, and the *Grade Level Expectations* (GLE), posted on the Internet on February 5, 2004. The GLE provides grade-level standards for each of the grades pre-K to 10 and a set of standards for the combined grades 11 and 12. The framework is scheduled for revisions once every 10 years; a revision of the 1997 *Framework* was not available at the time of this writing.

Louisiana's middling final score for 2005 is the result of documents that vary wildly in quality. The 1997 *Framework* was reviewed in 1998 and 2000 and judged among the worst of all states' standards, receiving an unambiguous "F" in both years. We have nothing to add to the previous evaluations concerning this dreadful document. Our comments here are focused on the 2004 *Expectations* document, which is generally solid, though the numerical scores provided here are based on evaluations of both documents (with somewhat greater emphasis on the later document).

2005 STATE REPORT CARD		
Louisiana		
Clarity: 2.00	C	
Content: 2.33	C	
Reason: 1.00	D	
Negative Qualities: 1.25	D	
Weighted Score: 1.78	Final Grade:	C
2000 Grade: F		
1998 Grade: F		

Standards for the elementary grades call for memorization of the basic number facts and facility in whole number computations—both positive features. However, there is no mention of the standard algorithms of arithmetic. Calculators are introduced in the third-grade standards, with the potential to undermine facility with whole number computations.

Poor Development of Concepts

The conceptual development of fraction arithmetic is problematic throughout. Students are not expected to understand the fundamental notion of equivalence of fractions until fifth grade, when they are asked to "Recognize, explain, and compute equivalent fractions for common fractions." Multiplication and division of fractions is not introduced until grade 7, but earlier grade standards—those for grades 5 and 6—implicitly assume an understanding of these operations. In grade 5, students

> *explain concepts of ratios and equivalent ratios using models and pictures in real-life problems (e.g., understand that 2/3 means 2 divided by 3).*

To fully understand that 2/3 equals 2 divided by 3 requires the concepts of multiplication and division of fractions. For the example given here, direct computation using the definitions of multiplication and division yields the result immediately: $2 \div 3 = 2 \times 1/3 = 2/3$. If a more informal understanding is intended here (such as partitioning a segment of length 2 into 3 congruent smaller segments and recognizing the length of each smaller segment as 2/3), this should be clarified. The following sixth-grade standards also rely on fraction multiplication and division:

> *Mentally multiply and divide by powers of 10 (e.g., 25/10 = 2.5; 12.56 x 100 = 1,256).*

> *Divide 4-digit numbers by 2-digit numbers with the quotient written as a mixed number or a decimal.*

> *Use models and pictures to explain concepts or solve problems involving ratio, proportion, and percent with whole numbers.*

In these three standards, multiplication and division of decimals is expected prior to the introduction (in the following year) of these operations for fractions. Since decimals represent fractions, the meaning of multiplication and division of decimals for sixth-graders is evidently left open. The emphasis here is then necessarily on procedures, with too little attention to mathematical reasoning.

Too Much Technology and Probability

The writing is sometimes difficult to understand, as in these standards for eighth and tenth grade respectively:

> *Estimate the answer to an operation involving rational numbers based on the original numbers.*

> *Identify and describe the characteristics of families of linear functions, with and without technology.*

With regard to this latter standard, it is unclear what the characteristics and families of linear functions might be, or how technology could be used in an appropriate way for whatever purpose is intended here. This is a common failing of the high school standards, which overemphasize technology.

Probability is introduced prematurely in first grade with this standard:

> *Appropriately use basic probability vocabulary (e.g., more likely to happen/less likely to happen, always/never, same as).*

Since probabilities are numbers between zero and one, it makes little sense to study probability before fractions are understood. And as with other states, data analysis and probability are overemphasized generally, with standards in these areas at all grade levels.

Maine

Reviewed: *Learning Results*, July 1997. The Maine mathematics standards are embodied in a single 13-page document entitled *Learning Results* dated July 1997. Eleven categories of standards are applied to each of the four grade bands: pre-K-2, 3-4, 5-8, and “Secondary Grades.”

2005 STATE REPORT CARD

Maine

Clarity: 1.17	D
Content: 1.17	D
Reason: 0.50	F
Negative Qualities: 2.75	B
Weighted Score: 1.35	Final Grade: D
2000 Grade: D	
1998 Grade: F	

Maine’s standards and its grade remain unchanged. Generally, these standards are vague and frequently open to a variety of interpretations. For example, at the elementary school level, students are to “explore the use of variables and open sentences to describe relationships” and “represent and describe both geometric and numeric relationships,” which provides almost no guidance to teachers. At the middle school level, students are to “identify patterns in the world and express these patterns with rules” and “demonstrate an understanding of inequalities and nonlinear equations.”

The first of the above quoted standards could be taken to describe all of physics, but here it is applied to grades 5-8. What are teachers expected to do with this? The second quoted standard leaves open what kinds of inequalities and nonlinear equations students are expected to “understand” and what it means to understand them.

At the high school level, students are to “describe the structure of the real number system and identify its appropriate applications and limitations.” The ways in which this sweeping instruction to teachers could be interpreted are innumerable.

In fairness, the Maine standards are supported by examples. However, most of them do not sufficiently illuminate the standards. For example, to illustrate some grade 5-8 geometry standards, the following example is given:

> *Collect magazine pictures of different styles of architecture and identify all the geometric figures and relationships seen in each building.*

There are other problems with these standards. For example, the reasoning standards for middle school are:

1. *Support reasoning by using models, known facts, properties, and relationships.*
2. *Demonstrate that multiple paths to a conclusion may exist.*

> *EXAMPLE: Prepare proposals for a fixed-height bridge and a draw bridge. Make recommendations after considering total cost, steepness of incline, traffic patterns, time of construction, etc.*

The activity suggested in the example is an aimless activity with little connection to mathematics that detracts from the standards.

Algebra receives insufficient attention at the high school level. The examples mention the quadratic formula, but there is no indication that students are expected to complete the square and understand the derivation of the quadratic formula. The word “proof” does not appear in these standards, and the only appearance of the word “theorem” is in an example for the high school standards that exhorts students to “discover and explore the distance formula using the Pythagorean Theorem.”

Maryland

Reviewed: *Mathematics Voluntary State Curriculum*, August 2003; *Algebra/Data Analysis and Geometry*. Maryland's Voluntary State Curriculum provides standards for each of the grades from pre-K to 8 and defines what students in those grades are expected to know or be able to do. The high school framework provides a minimal curriculum with the expectation that schools will augment those topics. At the time of this writing, high school standards were available only for Algebra/Data Analysis and Geometry.

2005 STATE REPORT CARD

Maryland

Clarity: 2.00	**C**
Content: 1.67	**D**
Reason: 1.50	**D**
Negative Qualities: 2.00	**C**
Weighted Score: 1.77	Final Grade: C

2000 Grade: **C**

1998 Grade: **F**

Maryland's unusual standards are very general, but then are followed by more specific grade level "Objectives," and then (starting in third grade) even more specific "Assessment Limits." For example, the following is a third-grade objective with an Assessment Limit:

> *Objective: Represent and analyze numeric patterns using skip counting backward.*
>
> *Assessment Limits: Use 10 or 100 starting with any whole number (0-1000).*

The Assessment Limit indicates the extent to which students may be tested on this objective on the state examination. In this case, third-graders may be given any starting number between 0 and 1000 (and presumably between 10 and 1000), and then be called upon to count backward from that number by 10s or by 100s. This example illustrates a general feature of the Maryland elementary and middle school standards: The Assessment Limits often restrict the range of objectives and could thus induce teachers to limit their teaching to the Assessment Limits.

In other cases, though, the objectives are so vague that assessment limits are indispensable for understanding what is intended. In eighth grade, for example, one objective is to "apply right angle concepts to solve real-world problems." The related Assessment Limit states, "Use the Pythagorean Theorem." This is the only explicit mention of the Pythagorean Theorem anywhere in the standards. Without this Assessment Limit, there would be no clear indication that the Pythagorean Theorem is part of the middle school curriculum.

Odd Objectives

And then there is "Process of Mathematics," a strand of exhortations that is repeated without change in grades pre-K to 8. Some of the objectives offer good advice, such as, "Identify the question in the problem." Others have little meaning, or are guilty of false doctrine when they direct students to "guess and check." Some objectives are inflated, e.g., "Identify mathematical concepts in relationship to life." Worse, since reading and writing are required to satisfy some of the objectives, they are inappropriate for pre-K and lower elementary standards.

Highlighted objectives in the document are tested in the "no calculator" section of the state exam, within the limits identified by the accompanying Assessment Limits. It is a positive feature of these standards that students are expected to carry out at least some computations without calculator assistance.

Memorization of basic facts is not explicitly called for in the elementary school grades; instead, students are to "demonstrate proficiency with addition and subtraction basic facts using a variety of strategies." Another shortcoming is that the standard algorithms of arithmetic are never even mentioned. When division students are finally expected to divide three-digit numbers by a one- or two-digit number in fifth grade, they are allowed to use calculators for the state assessment.

There are occasional curious leaps in the level of expectation in these standards. For example, through fifth grade, the development of area is restricted to rectangles. Then in sixth grade, students are abruptly expected to "estimate and determine the area of a polygon." There is no explicit expectation for students to understand a logical progression of formulas for areas of basic polygons by relating areas of triangles to areas of rectangles, parallelograms, and trapezoids in a coherent way. However, basic algebra skills are reasonably well developed in the lower grades, with expectations such as this first-grade task: "Find the missing number (unknown) in a number sentence. . . ." Likewise, word problems show up early and appear regularly.

Fraction and decimal arithmetic are not fully developed until the middle grades. A sixth-grade objective is "divide decimals," but students are not exposed to division of fractions until seventh grade. The possibility then exists that division of decimals is presented only as a procedure, without any conceptual framework to back it up. No standard or objective in any of the grades addresses irrational numbers in any way. A seventh-grade objective is, "Estimate pi using physical models," but there is no call for students to know the meaning of pi, and it is not until eighth grade that the objective "estimate and determine the circumference or area of a circle" is given.

There are other shortcomings in middle school geometry. Some of the objectives are ambiguous and confusing:

> *Objective: Identify and describe line segments.*
>
> *Assessment Limits: Use diagonal line segments.*

> *Objective: Identify or describe angle relationships.*
>
> *Assessment limits: Use perpendicular bisectors or angle bisectors.*

In the latter example, should students be able to bisect angles with a compass and straight edge or just recognize such bisections? If it is the latter, how are they to be recognized? Compass and straight edge are not mentioned, even where they should be, such as in this seventh-grade objective:

> *Objective: Construct geometric figures using a variety of construction tools.*
>
> *Assessment limits: Construct a perpendicular bisector to a given line segment or a bisector of a given angle.*

The high school course standards do not go significantly beyond what is expected by the end of eighth grade. The high school Algebra/Data Analysis document is more data analysis than algebra. One indicator states that "the student will interpret data and/or make predictions by finding and using a line of best fit and by using a given curve of best fit." This is college-level mathematics so whatever is intended here is inappropriate. The document explains that technology can be used when appropriate. This, of course, is not mathematics. The intention is merely for students to press buttons on a graphing calculator.

The algebra standards are weak. There is no mention of quadratic polynomials or methods for finding their roots. Nonlinear functions are mentioned, but not always appropriately:

> *The student will describe the graph of a non-linear function and discuss its appearance in terms of the basic concepts of maxima and minima, zeros (roots), rate of change, domain and range, and continuity.*

This standard belongs in a calculus course. Its placement in a standards document for beginning algebra is an example of inflation.

The geometry standards are also weak. Straight edge and compass are mentioned, as are two-columned proofs, but exactly what to do with them is left unclear. No specific theorems are to be proven.

Statistics and probability are overemphasized throughout the grades, and are sometimes too advanced. For example, in third grade, students are expected to make graphs of data using scaling before the appropriate mathematics (division) has been covered. The emphasis on patterns is excessive, with particularly ridiculous standards such as:

> *1st Grade: Recognize the difference between patterns and non-patterns.*

> *2nd Grade: Represent and analyze growing patterns that start at the beginning and show no more than 3 levels, and ask for the next level, using symbols, shapes, designs, and pictures.*

3rd Grade: Represent and analyze growing patterns using symbols, shapes, designs, or pictures.

4th Grade: Generate a rule for the next level of the growing pattern.

The pursuit of patterns in these standards is an end in itself with little connection to mathematics.

Massachusetts

Reviewed: *Massachusetts Mathematics Curriculum Framework*, November 2000; *Supplement to the Massachusetts Mathematics Curriculum Framework*, May 2004. The *Framework* provides standards for two-year grade spans from PreK-K to 11-12, and for the courses Algebra I, Geometry, Algebra II, and Pre-Calculus. In addition, the 2004 Supplement gives standards for the individual grades 3, 5, and 7 for the purpose of annual testing required by the No Child Left Behind Act. Each grade span includes extra standards under the heading, "Exploratory Concepts and Skills." These enrichment topics are not assessed by the state at the grade levels in which they appear, but some of them are also listed in later grade-level standards.

Massachusetts — **2005** STATE REPORT CARD

Clarity: 3.67	A
Content: 3.67	A
Reason: 2.00	C
Negative Qualities: 3.50	A
Weighted Score: 3.30	Final Grade: A
2000 Grade: **D**	
1998 Grade: **F**	

Massachusetts did its students a tremendous service in 2000 by jettisoning its old standards and substituting these clear, well-organized documents. They outline a solid and coherent program for mathematics education. The elementary grade standards are particularly strong. They require memorization of the basic number facts and facility with the standard algorithms of arithmetic. Students are expected to compute with and solve word problems involving fractions, decimals, and percents by the end of sixth grade. Rational number arithmetic and the field properties are thoroughly developed in the middle grades, with algebra and more advanced topics addressed by the high school standards.

Mixed Guidance on Technology

Technology plays a mixed role in these standards. A section of the *Framework*, "Guiding Principle III: Technology," begins with the declaration, "Technology is an essential tool in a mathematics education." The opening sentence of the final paragraph of the section is, "Technology changes what mathematics is to be learned and when and how it is learned." Both of these sweeping assertions overstate the importance of technology for K-12 mathematics.

On the other hand, this section also includes an important and refreshing caveat:

Elementary students should learn how to perform thoroughly the basic arithmetic operations independent of the use of a calculator. Although the use of a graphing calculator can help middle and secondary students to visualize properties of functions and their graphs, graphing calculators should be used to enhance their understanding and skills rather than replace them.

The Massachusetts standards deal admirably with technology in the elementary grades, but offer little guidance for its proper use in the higher grades. For example, *Exploratory Concepts and Skills* for grades 9 and 10 includes the suggested project, "Explore higher powers and roots using technology." Several standards include the ambiguous statement "use technology as appropriate," such as the following:

7.P.6 Use linear equations to model and analyze problems involving proportional relationships. Use technology as appropriate.

AI.P.11 Solve everyday problems that can be modeled using linear, reciprocal, quadratic, or exponential *functions. Apply appropriate tabular, graphical, or symbolic methods to the solution. Include compound interest, and direct and inverse variation problems. Use technology when appropriate.*

AII.P.8 Solve a variety of equations and inequalities using algebraic, graphical, and numerical methods, including the quadratic formula; use technology where appropriate. Include polynomial, exponential, and logarithmic functions; expressions involving the absolute values; and simple rational expressions.

Considering the diversity of teachers' opinions on the use of technology, the *Framework* would be improved if it clarified its directive to "use technology appropriately."

Inconsistent Reasoning

Standard AI.P.11 cited above illustrates the lack of specificity found in some of the higher grade-level standards. What "everyday problems" are intended here? What is a "reciprocal function"? And what are "tabular methods"? In a similar vein, a seventh-grade standard calls upon students to "solve linear equations using tables, graphs, models, and algebraic methods." How can linear equations be solved using tables? The appropriate methods for solving a linear equation in seventh grade are algebraic. If the solution of simultaneous linear equations is intended here, then graphical methods also play an important role, but this standard does not specify whether one or more linear equations are to be solved.

Mathematical reasoning is prominently featured. All Massachusetts standards are prefaced with the phrase, "Students engage in problem solving, communicating, reasoning, connecting, and representing as they: . . ." But the standards also go beyond this perfunctory exhortation. The following two standards for algebra and geometry respectively illustrate the incorporation of mathematical reasoning in the Massachusetts standards:

Use properties of the real number system to judge the validity of equations and inequalities, to prove or disprove statements, and to justify every step in a sequential argument.

Write simple proofs of theorems in geometric situations, such as theorems about congruent and similar figures, parallel or perpendicular lines. Distinguish between postulates and theorems. Use inductive and deductive reasoning, as well as proof by contradiction. Given a conditional statement, write its inverse, converse, and contrapositive.

The *Framework* also requires a comprehensive treatment of methods for finding the roots of quadratic polynomials in the algebra standards:

Find solutions to quadratic equations (with real roots) by factoring, completing the square, or using the quadratic formula. Demonstrate an understanding of the equivalence of the methods.

We interpret this to mean that students are expected to know how to derive the quadratic formula by completing the square, and to understand that the roots of a quadratic polynomial are given by the quadratic formula. However, we would prefer a clearer statement such as:

Find the roots of quadratic polynomials (with real roots) by factoring, by completing the square, **and** *by using the quadratic formula. Derive the quadratic formula by completing the square, and prove that the roots of a quadratic polynomial are given by the quadratic formula.*

There is no standard that explicitly requires students to see or understand a proof of the Pythagorean Theorem. The closest the *Framework* comes to this requirement is the following eighth-grade standard:

Demonstrate an understanding of the Pythagorean Theorem. Apply the theorem to the solution of problems.

What does it mean to "demonstrate an understanding of the Pythagorean Theorem"? Does it mean to understand the statement of the theorem? Or to understand a geometric interpretation of the theorem in terms of areas? Or, perhaps, even a proof? One can only guess.

Opportunities for the incorporation of mathematical reasoning are missed in standards that address topics in area, volume, and perimeter. Consider the following geometry standard:

Given the formula, find the lateral area, surface area, and volume of prisms, pyramids, spheres, cylinders, *and cones, e.g., find the volume of a sphere with a specified surface area.*

There is no requirement in any of the Massachusetts standards for students to understand how to derive or deduce any formula for area, perimeter, or volume of any geometric figure or solid. Surely, students can be expected to find the lateral surface area of prisms without being "[g]iven the formula."

Minor Problems

There are other problems with the Bay State's standards. As elsewhere, data analysis, statistics, and probability standards are overemphasized throughout the standards. This starts in pre-K and Kindergarten, where students are expected to construct bar graphs. In grades 1 and 2, students

decide which outcomes of experiments are most likely.

It makes no sense to teach probability to students before they have reasonable facility with fractions, since probabilities are, by definition, numbers between zero and one.

Data analysis, statistics, and probability standards are also inappropriately included among the standards for Algebra I and Algebra II. Among these standards is:

Approximate a line of best fit (trend line) given a set of data (e.g., scatterplot). Use technology when appropriate.

To develop the topic of lines of best fit properly is college-level mathematics, and to do it in other ways is not mathematics. Manipulation of polynomials is too restrictive:

Add, subtract, and multiply polynomials. Divide polynomials by monomials.

The four basic arithmetic operations should be performed with rational functions, not just with polynomials (or monomials). Requiring division by binomials would at least support a theorem addressed in the Pre-Calculus standards:

Relate the number of roots of a polynomial to its degree. Solve quadratic equations with complex coefficients.

In spite of these shortcomings, the Massachusetts math standards are among the best in the nation.

Michigan

Reviewed: *Michigan Curriculum Framework*, 1996; *Michigan Curriculum Framework: Mathematics including Teaching & Learning Activities*, 1998; *Grade Level Content Expectations, v. 6.04*, March 30, 2004. The Michigan standards are presented in a math framework augmented by sample activities and the more recent *Grade Level Content Expectations*. The *Framework* provides general content standards for three grade bands: Elementary, Middle School, and High School. The *Grade Level Content Expectations* provides standards for each of the grades K-8.

2005 STATE REPORT CARD

Michigan

Clarity: 2.17	C
Content: 1.67	D
Reason: 2.00	C
Negative Qualities: 2.50	B
Weighted Score: 2.00	Final Grade: C
2000 Grade: **F**	
1998 Grade: **F**	

The addition of the *Grade Level Content Expectations* in 2004 was a distinct improvement to Michigan's standards, though the state still has some distance to go. The *Framework* enunciates content standards so general that in some cases the same standard applies to all three grade bands, for example, this geometry standard: "Use shape, shape properties and shape relationships to describe the physical world and to solve problems." The sample activities aren't much better. In the case of this geometry standard, the following are among the suggested sample activities for students in the elementary, middle, and high school grades, respectively:

looking for shapes in advertising brochures, and writing about how the shapes create a pleasing graphic,

conducting open-ended investigations involving shapes, such as coloring maps or finding all the pentominoes and determining which pentominoes can be folded to make an "open box," and

using shape concepts to help make sense of observations in business, science, sports, and the environment. . . .

Additional comments on the Michigan *Framework* are available in the 1998 and 2000 Fordham reports at http://www.edexcellence.net/foundation/publication/publication.cfm?id=24#215.

The remainder of this report addresses the newer document, *Grade Level Content Expectations* (GLCE), which provides standards for each of the grades K-8, but our numerical scores of necessity reflect the influence of both documents, since only the *Framework* provides standards for the high school grades.

The GLCE is a frustrating mix of well-crafted and coherent standards on the one hand, and a carelessly written patchwork of topics on the other. Considerable care is given to the logical development of fraction arithmetic in these standards. Beginning in second grade, fractions are identified as points on a number line. The third-grade standards continue this development with well-crafted standards such as these:

Understand that any fraction can be written as a sum of unit fractions, e.g., 3/4 = 1/4 + 1/4 + 1/4.

Recognize that addition and subtraction of fractions with equal denominators can be modeled by joining and taking away segments on the number line.

This last standard offers a credible elementary level definition of fraction addition and subtraction. Standards in subsequent grades continue this coherent development of fractions. By sixth grade, students are expected to "add, subtract, multiply and divide positive rational numbers fluently." The parallel development of decimals is similarly well done.

The standards devoted to whole number arithmetic are less well-developed. Students are expected to memorize the addition facts, but they need only "solve the related subtraction problems fluently." Surprisingly, there is no requirement to memorize the multiplication tables or the corresponding division facts.

Some of the standards are overly restrictive, such as these fifth- and sixth-grade standards:

Add and subtract fractions with unlike denominators of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 100, using the common denominator that is the product of the denominators of the 2 fractions, e.g. . . .

Add, subtract, multiply, and divide integers between -10 and 10; use number line and strip models for addition and subtraction.

In the first of these standards, why the restriction on denominators? By fifth grade, students should be able to compute *1/2 + 1/13*, for example. In the second standard for sixth grade, the restriction to integers between 10 and -10 is completely artificial.

In eighth grade, irrational numbers are covered in two admirably clear standards:

Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals,

Understand that irrational numbers are those that cannot be expressed as the quotient of two integers, and cannot be represented by terminating or repeating decimals; approximate the position of familiar irrational numbers (e.g. $\sqrt{2}$ *,* $\sqrt{3}$ *, pi) on the number line.*

However, without the long division algorithm, not mentioned in this document, it is unclear how students are to achieve the understandings called for in these two standards. Moreover, a seventh-grade standard under the heading "Recognize irrational numbers" is nothing more than a calculator exercise: "Understand the concepts of square root and cube root, and estimate using calculators."

The algebra and geometry standards in the middle school grades are generally strong. Students are expected to know at least one proof of the Pythagorean Theorem and to use that theorem and its converse to solve problems. However, the algebra and geometry strands also include frustrating gaps. For example, a seventh-grade standard asks students to:

> *Use compass and straightedge to perform basic geometric constructions: the perpendicular bisector of a segment, an equilateral triangle, and the bisector of an angle; understand informal justifications.*

But how are students to "understand informal justifications"? No standard mentions the sufficient criteria for congruence of triangles (SSS, ASA, SAS). Yet, seventh-graders are expected to know the more sophisticated analogues of these congruence criteria to prove similarity of triangles:

> *Show that two triangles are similar using the criteria: corresponding angles are congruent (AAA similarity); the ratios of two pairs of corresponding sides are equal and the included angles are congruent (SAS similarity); ratios of all pairs of corresponding sides are equal (SSS similarity); use these criteria to solve problems and to justify arguments.*

The eighth-grade standards for probability are unnecessarily repetitive and should be edited.

Minnesota

Reviewed: *Minnesota Academic Standards, Mathematics K-12*, May 19, 2003. The document for mathematics provides grade-level benchmarks for each of the grades K-8, a set of benchmarks for the band of grades 9-11, and additional benchmarks for grades 11 and 12 for "students choosing more electives in mathematics or taking those electives at an earlier grade than their classmates."

2005 STATE REPORT CARD

Minnesota

Clarity: 2.00	C
Content: 1.67	D
Reason: 1.00	D
Negative Qualities: 2.00	C
Weighted Score: 1.67	Final Grade: D

2000 Grade: **F**

1998 Grade: **-**

Minnesota is on the right track with these recently revised standards, though the state still has a distance to travel. A marked improvement over the dreadful standards they replaced, the new math standards can nonetheless fairly be termed mediocre in most respects, with positive features undercut by inexplicable omissions and errors.

Happily, the elementary school standards require students to master the addition and multiplication facts for sums and products of single-digit numbers, and the related subtraction facts. (Oddly, though, there is no mention of the related division facts.) The Minnesota standards also make clear that students are expected to compute sums and differences of three-digit numbers by hand. However, there is no mention of regrouping (borrowing or carrying), nor of the important long division algorithm. The introduction in fifth grade of multiplication and division of decimals prior to any definition or explanation of multiplication or division of fractions in general is a content deficit, and shows a lack of attention to mathematical reasoning. What does it mean to multiply *3.2 x 3.4*, as prescribed by the fifth-grade standards, if fraction multiplication has not even been defined? Deficiency in fraction arithmetic continues in grade 6, where the benchmark for fraction arithmetic reads:

> *Add, subtract, multiply, and divide common fractions and mixed numbers as well as fractions where the common denominator equals one of the denominators.*

This standard has several defects. To begin, it should be broken into two standards, since it prescribes two related but distinct exercises. The restriction on denominators is not appropriate for multiplication and division of fractions. It is true that addition and subtraction of general fractions should be preceded by practice in computing sums when the common denominator is the denominator of one of the summands, but such a standard would be more appropriate in an earlier grade. Considering the explicit endorsement of calculator use beginning in grade 6, and the woefully insufficient grade 7 learning expectation (the student will "Add, subtract, multiply, and divide fractions and mixed numbers"), it appears that paper and pencil calculations with fractions are grossly underemphasized.

Patterns and Algebra

In grades K-5, the study of patterns appears to be an end in itself, with only weak connections to mathematics. Students create, identify, examine, describe, and extend repeating, growing, and shrinking patterns, where the patterns may be found in numbers, shapes, tables, and graphs.

The algebra standards for the middle grades are weak and redundant—a serious shortcoming, since the middle grades should build a foundation for deeper study of algebra in later years. Each set of standards for grades 6, 7, and 8 includes benchmarks calling upon students to understand the order of operations convention, including the respect of that convention when using calculators. A benchmark of this type appears yet again for grades 9-11. However, there is no mention of the distributive law until grades 9-11, and no explicit mention of the quadratic formula, much less its derivation, in any grade. The standards for grades 11 and 12 for more mathematically advanced students, do, however, require students to complete the square of quadratic polynomials and to identify complex roots of quadratic polynomials.

The geometry standards for K-8 include benchmarks addressing perimeter and area for basic two-dimensional figures, volume and surface area, classification of angles, triangles, and quadrilaterals, two-dimensional coordinate grids, and conversion of units of measurement. The Pythagorean Theorem is not mentioned until grades 9-11, long after it ought to have been introduced. In those grades, students are expected to know and use a variety of theorems of geometry, but there is no explicit mention of proofs.

Editors Needed

The document would benefit from careful editing by someone who knows mathematics. The writing is sometimes obscure, garbled, or ungrammatical, as below.

> *Use fractions and decimals to solve problems representing parts of a whole, parts of a set, and division of whole numbers by whole numbers in real-world and mathematical problems.*

It cannot be that the problems represent "parts of a whole," though fractions might. The phrase "length of sides" is ambiguous in this benchmark:

> *Identify, describe and classify two-dimensional shapes according to number and length of sides and kinds of angles.*

Finally, there are standards so vague that teachers may only guess as to their meaning, as in, "Use mathematical language to describe a set of data."

Mississippi

Reviewed: *Mississippi Mathematics 2000 Framework*, as contained in the *Mathematics Instructional Intervention Supplement*, has standards for each grade K-8 and 13 high school courses (Pre-Algebra, Transitions to Algebra, Algebra I, Geometry, Survey of Mathematical Topics, Algebra II, Advanced Algebra, Pre-Calculus, Trigonometry, AP Calculus, Discrete Mathematics, Probability and Statistics, and AP Statistics).

In 2000, the Fordham reviewers evaluated a draft of the Mississippi *Framework* that later underwent significant revision by the state department. Our present review evaluates the final draft of the 2000 math standards as distributed to districts and published on the state website. Simply put, the editing the state did to its standards after our 2000 review was published was a disaster.

The first task in reviewing Mississippi's math standards is sorting out from among the scattered documents exactly *what* to review. The *Framework* involves five strands of "competencies" and "suggested teaching objectives" for each grade or course. The competencies

2005 STATE REPORT CARD		
Mississippi		
Clarity: 1.33	D	
Content: 2.00	C	
Reason: 1.00	D	
Negative Qualities: 2.00	C	
Final Weighted Score: 1.67	Final Grade:	D
2000 Grade: **A**		
1998 Grade: **B**		

are deliberately broad in order "to allow school districts and teachers flexibility" in creating curricula. For that same reason, the suggested teaching objectives, while often specific, are optional. In third grade, for example, one finds the vaguely worded competency, "Develop the process of measurement and related concepts," followed by seven *optional* teaching objectives that include specific tasks such as "convert between pints, quarts, and gallons." As a result of this format, little content is actually required to be taught in Mississippi schools.

Mississippi also provides a version of the *Framework* for the use of teachers of grades K-8. This *Mathematics Instructional Intervention Supplement* makes a clearer, albeit different, statement about expectations for those grade levels. As explained in its introduction, the *Supplement* has three components: Benchmarks (what students should know and be able to do to meet mandated competencies), Assessments (informal assessments to determine if the benchmarks are being met), and Instructional Intervention Strategies (suggested classroom activities). Because the benchmarks are the same as the "suggested teaching objectives" of the *Framework*, this effectively means that those optional objectives will be assessed (although that is never explicitly stated). For that reason, we reviewed the Mississippi standards as they are presented in the *Mathematics Instructional Intervention Supplement*.

Mississippi's benchmarks vary widely in clarity, definiteness, and testability. While the majority of benchmarks are reasonably clear and precise, many others are too general to give adequate guidance to teachers. What exactly is covered by the third-grade standard "Compares metric measurements to English measurements"? Many standards ambiguously require students to "explore," "investigate," "model," "discuss," or "demonstrate." Such standards are often difficult to interpret. We list a few representative examples:

> *Grade 2*
> *43) Identifies, discusses, and draws representations of equivalent fractions through one-third.*
>
> *Grade 3*
> *Uses multimedia resources to investigate and solve word problems.*
>
> *33) O - Incorporates appropriate technology and manipulatives to explore basic operations of whole numbers, fractions, mixed numbers, and decimals.*
>
> *Algebra I*
> *3.b. Model properties and equivalence relationships of real numbers.*
>
> *Geometry*
> *4.d. Explore how change in perimeter results in a change in area.*

This last example is not only vague, but misleading, as there is no functional relationship between area and perimeter for polygons. The above examples also demonstrate an over-reliance on technology and manipulatives in the Mississippi standards. The *Framework* makes clear its intention with regard to technology in the following statement:

> *The Mississippi Department of Education strongly encourages the use of technology in all mathematics classrooms.*

Calculators are introduced explicitly in first grade, when the student "explores and explains patterns of addition and subtraction with and without the use of a calculator." While third-graders are expected to remember multiplications up to *5 x 5*, all other standards referring to basic facts ask students to be able merely to compute, not recall, those facts, and common strategies that make it easy to learn those facts are not mentioned.

Slow Development

The standards are excessively incremental regarding place value, addition, and subtraction. Students read and write one-digit numbers in Kindergarten, two-digit numbers in first grade, and proceed to nine-digit numbers in fifth grade and twelve-digit numbers in sixth grade. By fifth grade, students add and subtract "nine-digit whole numbers with and without regrouping." Explicit expectations are provided for whole number, fraction, and decimal computations throughout the grades, but there is no mention of the standard algorithms of arithmetic. Instructional Intervention Strategies lessons make use of less efficient algorithms, and it is unclear to what extent students may use calculators.

The standards on probability are vague and are developed very slowly. The fourth-grade standard "Investigate the concepts of probability" and similar standards for grades 2, 3, and 5 give no hint about what students are expected to learn, know, or do. In sixth grade, we finally get a definite standard: "[Use] probability to predict the outcome of a single event and express the result as a fraction or decimal." This standard is repeated in abbreviated form in seventh grade. Thus, after *six years* of studying probability, students can determine the probability of a single event (e.g., of getting a two upon rolling one die) and *nothing else*. In contrast, coherent curricula hold off on introducing probability until middle school, and then proceed quickly by building on students' knowledge of fractions and ratios.

Ratios, percents, fractions, and prime numbers are covered well in grades 6 and 7, but equations are not graphed until the eighth grade. Some standards are poorly worded or nonsensical, such as the sixth-grade benchmark, "Explore the relationship between integers," and the seventh grade benchmark, "use patterns to develop the concept of exponents."

The standards outline a solid Pre-Algebra course, but the content is at the level of grade 7 or 8, not high school. That Algebra I course covers linear equations and polynomials well, but barely mentions quadratic equations. Instead of specific standards about solving quadratic equations, there is only the vague and mathematically incorrect standard: "Investigate and apply real number solutions to quadratic equations algebraically and graphically." Formulas are apparently to be "experimentally verified rather than derived through reasoning."

The standards for Geometry include many aspects of a good high school course, but are far too vague. There is a single standard that requires students to "develop and evaluate mathematically arguments and proofs," but that injunction is not specific enough to be meaningful or useful (*what* are students expected to be able to prove?) and is undermined by the fact that the proof of the Pythagorean Theorem is not even mentioned.

The standards for Algebra II are good, but include some vague directives, e.g., "Explore and investigate solutions to compound and absolute value inequalities to include interval notation" and "Explore and describe the complex number system." The single standard on Data Analysis, "use scatter plots and apply regression analysis to data," is not linked to the rest of the course. Advanced Algebra handles series and conic sections well, but omits important topics such as limits, the Fundamental Theorem of Algebra, and the number *e*.

Missouri

Reviewed: Missouri has four tiers of standards, the first being the *Show Me Standards*, January 18, 1996. Next, *Missouri's Framework for Curriculum Development In Mathematics K-12*, October 7, 2003, is designed to help school districts shape their curricula according to the standards. The *Framework Annotations* (same date) features what is to be assessed on the state assessments. Lastly, the *Mathematics Grade-Level Expectations* provides grade-by-grade content standards (same date). An additional document, *Achievement Level Descriptors: Mathematics*, August 26, 2003, provides further elaboration.

Missouri has undertaken multiple revisions and additions to its statewide academic standards in the past several years. One would hope that the *Show Me* math standards would now deserve a much higher mark than the failing grade they earned in two earlier reviews. No such luck. The standards remain so general as to be almost meaningless—and when they are specific, the content is consistently below grade level.

Missouri	2005 STATE REPORT CARD	
Clarity: 0.67	F	
Content: 0.33	F	
Reason: 1.00	D	
Negative Qualities: 0.50	F	
Weighted Score: 0.57	Final Grade:	F
2000 Grade: F		
1998 Grade: F		

For example, Standard 4 (out of a total of six) says:

> *In Mathematics, students in Missouri public schools will acquire a solid foundation which includes knowledge of patterns and relationships within and among functions and algebraic, geometric, and trigonometric concepts.*

The *Framework* is similarly vague, and implicitly compromises mathematical content by its organization into chapters beginning with *Problem Solving*, *Communications*, *Reasoning*, and *Connections*. *Number Sense* and other content topics are relegated to the end of the document. Actual standards cover the grade bands K-4, 5-8, 9-12, and are organized into three columns: "What All Students Should Know," "What All Students Should Be Able To Do," and "Sample Learning Activities." The guidelines have a tendency toward inflation, such as:

> *Evaluate the logic and aesthetics of mathematics as they relate to the universe.*

> *Use paper folding activities and/or computer technology to deduce properties and relationships between figures (such as exploring the relationships of opposite sides, opposite angles, diagonals of the quadrilateral family, relationships between angles, chords of a circle, etc.). Develop a simple deductive system using these relationships.*

> *Use the concept of recursion in mathematics to solve application problems (e.g., compound interest, depreciation, radium decay, maximum storage in the least amount of space, fractals).*

The document *Framework Annotation* promises "annotations that should be useful in understanding state and local responsibilities in assessing curriculum at the fourth, eighth, and tenth grade levels," but offers little content.

Almost-Standards

Mathematics Grade-Level Expectations comes closest to providing credible standards. On the positive side, the document asks students to "demonstrate fluency" with addition and multiplication in the second and fourth grades, respectively. However, there is no explicit requirement that students learn the standard algorithms of arithmetic. Generalizations and vague wording continue to be problems—e.g., "use real numbers to solve problems" (in tenth grade), and "model problem situations, using representations such as graphs, tables, or number sentences" (in fourth grade). Other standards are just incoherent, for example:

> *Describe, classify, and generalize relationships between and among types of a) 2-dimensional objects and b) 3-dimensional objects using their defining properties including • Pythagorean Theorem • cross section of a 3-dimensional object results in what 2-dimensional shape (grade 8).*

Overall, the Missouri grade-level expectations lag behind those of the better state standards by a year or more. For example, students are not expected to distinguish integers as even or odd until fourth grade. The number line makes its first appearance in sixth grade. Students are not expected to add fractions until sixth grade, multiplication and division of fractions does not appear until seventh grade, negative fractions are not introduced until eighth grade, and irrational numbers not until ninth. The "effects of parameter changes on linear functions" are not introduced until ninth grade. The standard "use a variety of representations to demonstrate an understanding of very large and very small numbers" appears as a tenth-grade standard (scientific notation should be introduced in the sixth or seventh grade). Quadratic polynomials are not mentioned until tenth grade, and there is no mention of the quadratic formula or completing the square at all. There are several inappropriate standards in tenth, eleventh, and twelfth grades that call upon students to "analyze quadratic functions [as well as exponential, rational, and other nonlinear functions] by investigating rates of change." But since there is no mention of derivatives in these standards, it is unclear how students can accomplish such analyses.

Montana

Reviewed: *Standards for Mathematics*, October 1998. Montana provides standards for spans of grades ending in grades 4, 8, and 12.

2005 STATE REPORT CARD

Montana

Clarity: 1.00	D
Content: 1.00	D
Reason: 0.00	F
Negative Qualities: 2.00	C
Weighted Score: 1.00	Final Grade: D

2000 Grade: **D**

1998 Grade: **F**

Montana remains stuck at the low end of our grading scale with these unrevised standards, which remain strikingly indefinite. Their language is so vague as to be useless in some cases. Consider, for example, this eighth-grade standard:

> *Recognize and investigate the relevance and usefulness of mathematics through applications, both in and out of school.*

Throughout, there is no clear indication that students are expected to memorize the single-digit arithmetic facts, learn the standard algorithms of arithmetic, the quadratic formula, or understand a proof of the Pythagorean Theorem. Students in the fourth grade or below are called upon to

> *select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, and computer.*

Allowing young students to use calculators when they think it is appropriate calls into question the mastery of arithmetic in elementary school.

Too much emphasis is given to data analysis, probability, and statistics relative to algebra and other topics. The high school algebra standard, "use algebra to represent patterns of change," is nearly vacuous. A ray of hope is offered by this high school standard:

> *Solve algebraic equations and inequalities: linear, quadratic, exponential, logarithmic, and power.*

However, even that hope is dampened by the possibility that this standard could be addressed largely through the use of graphing calculators. There is no indication to the contrary.

Nebraska

Reviewed: The Content Standards are listed in the appendix of Nebraska Department of Education, RULE 10, Regulation and Procedures for the Accreditation of Schools, Title 92, Nebraska Administrative Code, Chapter 10, 2004. Nebraska provides standards for grade spans K-1, 2-4, 5-8, and 9-12.

2005 STATE REPORT CARD

Nebraska

Clarity: 1.72	D
Content: 1.28	D
Reason: 0.67	F
Negative Qualities: 2.17	C
Weighted Score: 1.42	Final Grade: D

2000 Grade: **C**

1998 Grade: **F**

Nebraska's new math standards are no improvement. Overall, their content is sketchy. There is no clear indication that students are expected to memorize the single-digit arithmetic facts, learn the standard algorithms of arithmetic, the quadratic formula, or understand a proof of the Pythagorean Theorem. Much of the writing is awkward and vague. There is heavy reliance on calculators throughout.

Far too much time and attention are devoted to probability and data analysis (forty percent of the twelfth-grade standards). Many of these standards are vague, for example: "Justify the chosen sampling techniques."

Nearly all of the measurement standards involve actually measuring, as opposed to doing calculations with measurements. In fourth grade, these are rigorous, though in places inappropriately ambitious ("Estimate and accurately measure capacity to the nearest milliliter"), and require equipment (scales, graduated cylinders) not common in elementary schools.

The standards that mention conversions include the phrase "given conversion factors"—which turns a conversion problem that tests knowledge of conversion factors into a multiplication problem that is far too simple for the eighth and twelfth grades. Conversion within the metric system (important for developing the concept of place value) does not appear until eighth grade, long after it should.

The geometry standards are a peculiar mix, with the trivial and the substantive given equal billing. Too many standards involve simply naming shapes (e.g.: "Identify, describe, compare, and classify . . . polygons, circles, etc.)," with none of these shapes actually defined. Congruence and similarity, which are fundamental to conceptual understanding, reasoning, and problem solving in geometry, appear only as words on a list, without any indication about their use.

Standards for area and volume are given, but one standard contains the peculiar wording "given formulas for volume" of solids. Problems involving angle measures (e.g., for isosceles triangles and parallelograms) are never mentioned. Transformations are introduced with no apparent goal. The twelfth-grade geometry standards completely lack substance, save one, which involves introductory material on trigonometry. Finally, the treatment of algebra is weak by the end of grades 8 and 12.

Nevada

Nevada Mathematics Standards, February 25, 2003, provides standards for each of the grades K-8 and for grade 12. *Performance Level Descriptors: Mathematics* describes what it means for students in grades 2, 3, 5, 8, and 12 to exceed, meet, approach, or fall below expectations associated with content standards.

2005 STATE REPORT CARD

Nevada

Clarity: 2.17	C
Content: 1.33	D
Reason: 1.50	D
Negative Qualities: 2.50	B
Weighted Score: 1.77	Final Grade: C

2000 Grade: C

1998 Grade: -

Nevada's standards, revised in 2003, are, on the whole, mediocre. Students in the elementary grades are expected to memorize the basic number facts, and calculators play only a minor role throughout—two positive features. Whole number and decimal arithmetic is developed through fifth grade, culminating with "Multiply and divide multi-digit numbers by 2-digit numbers, including strategies for powers of 10," a solid expectation. However, the arithmetic standards do not require students to learn and understand the conventional algorithms of arithmetic, including the important long division algorithm.

There is a lack of coherence in the development of fraction arithmetic. For example, sixth-graders "Read, write, add, subtract, multiply, and divide using decimals, fractions, and percents." This standard, requiring facility with the arithmetic of positive rational numbers, appears abruptly in Nevada's standards. Multiplication and division of fractions are not explicitly mentioned in earlier grade-level standards—indeed, are not explicitly addressed in *any* grade-level standard. It is jarring to contrast the computational facility demanded by the above standard with the lower expectations of another sixth-grade standard: "Use models and drawings to identify, compare, add, and subtract fractions with unlike denominators; use models to translate among fractions, decimals, and percents." If sixth-graders are using manipulatives and drawings to add and subtract fractions, what methods are they expected to use to carry out a computation like 2/3 divided by 2.15?

In the Patterns, Functions, and Algebra strand, Nevada's standards emphasize "patterns," without a clear description of the kinds of patterns to be studied, a fault that these standards share with those of many other states. For example, in fourth grade, students are asked to "identify, describe, and represent numeric and geometric patterns and relationships." This is so indefinite that teachers must guess its intended meaning.

Algebra is poorly developed even by eighth grade, where it is limited to linear equations and the addition of binomials. The language in the standards is sometimes inept, e.g., "Model, identify, and solve linear equations and inequalities; relate this process to the order of operations" or "solve simple linear equations and connect that process to the order of operations." "Order of Operations" signifies a tool of many uses, as seen also in this fifth-grade standard: "Use order of operations to solve problems."

The twelfth-grade standards—the only high school standards—are pitched at a low level. There is no reference to proof, and the standards lack a systematic development of quadratic polynomials. Many of the high school standards really belong in the middle grades, for example, "Convert between customary and metric systems; convert among monetary systems," and "select and use measurement tools, techniques, and formulas to calculate and compare rates, cost, distances, interest, temperatures, and weight/mass."

There are also isolated standards for Problem Solving, Mathematical Communication, Mathematical Reasoning, and Mathematical Connections that are not part of the content standards. These offer little insight into how teachers might integrate these important topics into the content standards themselves.

New Hampshire

Reviewed: *K-12 Mathematics Curriculum Framework*, February 1995; *Addenda, Grades K-3, 4-6, and 7-10*, 1994, 1995, 1996; *Draft K-8 New Hampshire and Rhode Island Local and NECAP Grade Level Expectations* (GLEs), June 6, 2004.

2005 STATE REPORT CARD

New Hampshire

Clarity: 1.17	D
Content: 0.67	F
Reason: 0.00	F
Negative Qualities: 1.00	D
Weighted Score: 0.70	Final Grade: F
2000 Grade: C	
1998 Grade: C	

This year, New Hampshire and Rhode Island jointly implemented grade-level expectations associated with the New England Common Assessment Program used in both states (plus Vermont) in grades 3-8. They are not an improvement on New Hampshire's already mediocre *Framework*.

The *Framework* calls upon students to "explore" a variety of topics, use manipulatives, study and extend patterns, and use technology, with little discussion of the goals that lie at the end of these tasks. For example, one first-grade standard reads, "Provide opportunities for children to explore the relationship among pennies, nickels, and dimes." It is unclear what exactly students are expected to explore, what skills they are expected to acquire, and what knowledge they are expected to gain.

While some concessions are made to pencil and paper work and mastery of basic arithmetic facts, the emphasis on calculators in the *Framework* is extreme at all grade levels. In Kindergarten, teachers are directed to "allow students to explore one-more-than and one-less-than patterns with a calculator." In first grade, calculator standards include:

> *Have students explore patterns and place value concepts using a calculator. (What is ten more than 23?)*
>
> *Have students use calculators to explore the operation of addition and subtraction.*
>
> *Have children use calculators to find sums and differences.*

Second-grade standards include:

> *Have students explore patterns and place value concepts using a calculator.*
>
> *Build on children's skill with the calculator to explore number patterns and sequences.*

The emphasis on calculators increases in third grade when students and teachers are guided to:

> *Explore the multiplication facts with a calculator, examine patterns, make conjectures, and discuss children's findings.*
>
> *Have students continue to use calculators to explore ever-more sophisticated number patterns and sequences.*
>
> *Have students use manipulatives and calculators to explore statistics such as the median and mean.*

Manipulatives and Algebra

Physical models and manipulatives are emphasized by the *Framework* at all grade levels, at the expense of abstract reasoning. A sixth-grade standard reads, "Given a pair of fractions, determine which is larger by using physical models or illustrations." By the end of tenth grade, students are to "use physical models to represent rational numbers." There is no indication that students are expected to learn the standard algorithms of arithmetic. This has a negative impact on the level of mathematical reasoning that children can achieve from these standards. For example, a grade 4-6 standard is, "Demonstrate an understanding of the periodicity of numbers." This is unclear, but if "periodicity of numbers" refers to repeating blocks in the decimals for rational numbers, the tool for achieving this understanding–long division–is nowhere to be found in these standards.

The New Hampshire *Framework* standards for algebra are weak. "Linear" is the only specific reference to a type of equation. In grades 7-12, students are to "solve equations and inequalities in one or two variables, by informal and formal algebraic methods," but "concrete materials, tables, or graphs" and "trial and error" are the only specific references to methods for solving equations. Properties of equalities (or inequalities) are never mentioned. Graphing calculators or graphing software are the only methods mentioned for solving a system of linear equations. Even students in the higher grades (7-12) are expected to "Perform polynomial operations with manipulatives."

The standards for trigonometry and geometry are also vague and weak. Trigonometry barely appears in these standards, and in geometry one finds standards like "Explore the relationship among definitions, postulates, and theorems," with no further elaboration.

GLEs: No Improvement

The GLEs make no reference to calculators, but like the *Framework*, they overemphasize manipulatives. Students are not required to memorize the basic number facts, or to use or understand the standard algorithms of arithmetic.

The GLEs frequently suffer from convoluted writing, as illustrated by this fifth-grade standard:

> *M5:1 Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from 0 to 9,999,999 through equivalency, composition, decomposition, or place value using models, explanations, or other representations; positive fractional numbers (proper, mixed number, and improper) (halves, fourths, eighths, thirds, sixths,*

*twelfths, fifths, or powers of ten [10, 100, 1000]), decimals (to thousandths), or benchmark percents (10%, 25%, 50%, 75% or 100%) as a part to whole relationship in area, set, or linear models using models, explanations, or other representations.**

The asterisk in the last line references a footnote that places confusing restrictions on the rational numbers that students consider:

> **Specifications for area, set, and linear models for grades 5-8: Fractions: The number of parts in the whole are equal to the denominator, a multiple of the denominator, or a factor of the denominator. Percents: The number of parts in the whole is equal to 100, a multiple of 100, or a factor of 100 (for grade 5); the number of parts in the whole is a multiple or a factor of the numeric value representing the whole (for grades 6-8). Decimals (including powers of ten): The number of parts in the whole is equal to the denominator of the fractional equivalent of the decimal, a multiple of the denominator of the fractional equivalent of the decimal, or a factor of the denominator of the fractional equivalent of the decimal.*

Mathematical topics in the GLEs are poorly organized. In some cases the ordering of topics from one grade to the next makes no sense. Consider, for example, these fourth-and fifth-grade Expectations:

> *Demonstrates conceptual understanding of perimeter of polygons, and the area of rectangles, polygons, or irregular shapes on grids using a variety of models, manipulatives, or formulas. Expresses all measures using appropriate units.*

> *Demonstrates conceptual understanding of perimeter of polygons, and the area of rectangles or right triangles through models, manipulatives, or formulas, the area of polygons or irregular figures on grids, and volume of rectangular prisms (cubes) using a variety of models, manipulatives, or formulas. Expresses all measures using appropriate units.*

Fourth-graders "demonstrate conceptual understanding of . . . the area of polygons [*sic*]," while fifth-graders "demonstrate conceptual understanding" of right triangles. But triangles are polygons, and fourth-graders should understand how to find areas of rectangles and triangles before finding areas of more complicated polygons.

The only reference to slopes and linear functions in the GLEs for eighth grade is in this standard:

> *M(F&A)–8–2 Demonstrates conceptual understanding of linear relationships (y = kx; y = mx + b) as a constant rate of change by solving problems involving the relationship between slope and rate of change; informally and formally determining slopes and intercepts represented in graphs, tables, or problem situations; or describing the meaning of slope and intercept in context; and distinguishes between linear relationships (constant rates of change) and nonlinear relationships (varying rates of change) represented in tables, graphs, equations, or problem situations; or describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant and varying rates of change.*

The emphasis on "varying rates of change" for nonlinear functions is misplaced. That subject is better treated in a calculus course. Eighth-graders could spend their time more profitably by learning that the slope m and y-intercept b in the equation $y = mx + b$ can be determined from the coordinates of any pair of points on its graph—one more example of the misguided approach of this document.

New Jersey

Reviewed: *New Jersey Core Curriculum Content Standards for Mathematics*, 2002; *Mathematics Curriculum Framework*, 1996; *Questions and Answers Related to the Revised Core Curriculum Content Standards in Mathematics*, July 2, 2002. The New Jersey standards provide grade-level benchmarks for band of grades K-2, each of the grades 3-8, and standards for the end of grade 12. The standards are intended to outline a core curriculum suitable for all or nearly all students. The 1996 *Framework* provides supplementary material to the standards, including pedagogical advice. Another resource available to New

Jersey teachers is *Questions and Answers Related to the Revised Core Curriculum Content Standards in Mathematics*, July 2, 2002.

New Jersey	**2005** STATE REPORT CARD	
Clarity: 2.17	**C**	
Content: 1.17	**D**	
Reason: 0.50	**F**	
Negative Qualities: 0.75	**F**	
Weighted Score: 1.15	Final Grade:	D
2000 Grade: **C**		
1998 Grade: **C**		

New Jersey did itself no favors by revising its already middling math standards in 2002; it has dropped a full letter grade in our review. These standards do have some positive features, though. They are generally straightforward and clear. The field properties of rational and real numbers are well developed. Counting principles are carefully developed. The geometry standards for grades 2-8 are well written. Memorization of the basic arithmetic facts is explicitly required of elementary school students, and they are expected to carry out some whole number computations by hand.

However, use of the standard algorithms of arithmetic is not required, and hand calculation is undermined by standards that require the use of calculators at all grade levels. The 2002 "Questions and Answers" document makes clear that state mathematics assessments allow even elementary school students access to calculators *during state exams*:

> *Question: "Under the mathematical processes standard, indicator 4.5F4 says that students will 'use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions.' For what grade levels is this a reasonable expectation? Some teachers claim that they do not let their students use calculators until grade five or six, thinking that this will force them to become proficient at pencil-and-paper computation."*
>
> *Answer: "Calculators can and should be used at all grade levels to enhance student understanding of mathematical concepts. The majority of questions on New Jersey's new third- and fourth-grade assessments in mathematics will assume student access to at least a four-function calculator. Students taking any of the New Jersey Statewide assessments in mathematics should be prepared to use calculators by regularly using those calculators in their instructional programs. On the assessments, students should be permitted to use their own calculators, rather than the school's calculators, if they so choose. . . ."*

The same document explains that students will be examined partially on their understanding of manipulatives:

> *Several of the questions on the mathematics assessments will assume student familiarity with various commonly used manipulatives, including but not necessarily limited to the following: Base ten blocks, Cards, Coins, Geoboards, Graph paper, Multi-link cubes, Number cubes, Pattern blocks, Pentominoes, Rulers, Spinners, and Tangrams.*

These directives are not the excesses of an isolated document. The New Jersey *Framework* lists among its goals the incorporation of calculators into the early grades and the integration of manipulatives, normally reserved for the elementary grades, into high school, as indicated in this passage:

> *Young children find the use of concrete materials to model problem situations very natural. Indeed they find such modeling more natural than the formal work they do with number sentences and equations. Older students will realize that the adults around them use calculators and computers all the time to solve mathematical problems and will be prepared to do the same. Perhaps more challenging, though, is the task of getting the "reverse" to happen as well, so that technology is also used with young children, and the older students' learning is enhanced through the use of concrete models. Such opportunities do exist, however, and new approaches and tools are being created all the time.*

The *Framework* adds that "algebra tiles are used to represent variables and polynomials in operations involving literal expressions" for high school students.

This agenda is fundamentally anti-mathematical. Mastery of basic skills is essential to learning more advanced topics. Manipulatives can be effective pedagogical tools in the early grades, but ultimately the power of mathematics lies in its abstract nature. Promoting algebra tiles in place of the more powerful and abstract distributive property in the high school grades is an impediment to learning mathematics, not an aid.

Incomplete and Inappropriate Content

Moving to specific content, the treatment of algebra in high school is weak. There is no mention of solving two or more linear equations simultaneously by algebraic methods, of algebraic manipulations of rational functions, or of completing the square for quadratic polynomials. The treatment of trigonometry and conic sections is skimpy, and there is no mention of complex numbers.

Displacing such foundations, a strand of standards is devoted to "Discrete Mathematics—Vertex-Edge Graphs and Algorithms." In second grade, this strand includes the standard, "Play simple two-person games (e.g., tic-tac-toe) and informally explore the idea of what the outcome should be." It continues into the high school grades with a focus on graph theory. Also deviating from mainstream topics are standards for middle and high school students devoted to fractals and tessellations.

Further compromising middle and high school standards is a premature focus on topics more appropriately reserved for calculus courses. These include optimization problems, studying "slope of a line or curve," continuity, and monotonicity of functions, all with a heavy reliance on graphing technology.

New Mexico

Reviewed: *Mathematics Content Standards, Benchmarks, and Performance Standards*, June 2002. New Mexico provides standards for each of the grades K-8 and a single set of standards for grades 9-12. The grade 9-12 standards section includes a subsection with "Guides for Further Study" in algebra and geometry for more advanced students.

2005 STATE REPORT CARD

New Mexico

Clarity: 3.00	B
Content: 2.67	B
Reason: 2.00	C
Negative Qualities: 3.00	B
Weighted Score: 2.67	Final Grade: B
2000 Grade: **F**	
1998 Grade: **F**	

New Mexico deserves accolades for strong improvements in its statewide math standards since our last evaluation, when the state received an "F." Though not perfect, the new standards are well organized, coherent, and feature solid—though not stellar—coverage of important content.

In the early grades, the base ten structure of the number system is systematically developed. Elementary grade students are also expected to understand and use the standard algorithms of arithmetic, including the long division algorithm with two-digit divisors.

The arithmetic of fractions, decimals, and percents is thoroughly covered in the upper elementary and middle school grade standards. The middle school grades include many standards requiring students to solve problems in arithmetic, algebra, and geometry, a strong feature of these standards.

The standards for grades 9-12 cover a broad range of topics in algebra, geometry, trigonometry, and probability and statistics. Properties of, and calculations with, linear, quadratic, higher-degree polynomials, and rational functions are well developed. Geometric proofs, including proofs by contradiction, are required in the geometry standards. These standards outline a credible course of study for secondary students, with many opportunities for practice in problem-solving, including use of the quadratic formula, the Pythagorean Theorem, and its converse.

Content coverage is generally strong, despite shortcomings. The expectations with regard to memorizing the basic number facts are ambiguous. In third grade, students "use strategies (e. g., *6 x 8* is double *3 x 8*) to become fluent with the multiplication pairs up to *10 x 10*." In fourth grade, students

> *demonstrate multiplication combinations through 12 x 12 and related division facts, and use them to solve problems mentally and compute related problems (e. g., 4 x 5 is related to 40 x 50, 400 x 5, and 40 x 500).*

What does "demonstrate" mean in this context? Perhaps the authors intended for students to memorize the basic number facts, but the standards do not explicitly call for memorization, and would be stronger if they did.

Coherence and Clarity

There is a lack of coherence in the standards that address the arithmetic of fractions in the middle grades (5-8). For example, it is unclear whether students are expected to know how to divide fractions by sixth or seventh grade. On the one hand, under the 5-8 benchmark, "Understand the meaning of operations and how they relate to one another," sixth-graders are to "explain and perform addition, subtraction, and multiplication with fractions and mixed numerals." Sixth-graders are evidently not expected to know how to divide fractions. Only in seventh grade (and beyond) are students required to "add, subtract, multiply, and divide rational numbers. . . ." Division of fractions arises for the first time in seventh grade within this benchmark. However, under the 5-8 benchmark, "Compute fluently and make reasonable estimates," sixth-graders are to "compute and perform multiplication and division of fractions and decimals and apply these procedures to solving problems." So, which is it? We much prefer that students learn this important operation in the earlier grade, but the New Mexico standards are unclear on this issue.

In a similar vein, fifth-graders are expected to "compute a given percent of a whole number." But multiplication of decimals appears for the first time in the grade 6 standard, "Explain and perform . . . addition, subtraction, multiplication, and division with decimals." Multiplication of fractions appears first for grade 6 students. Fifth-graders are not expected to know how to multiply fractions (including decimals), yet they are expected to compute a given percent of a whole number.

Throughout, these standards overemphasize the importance of patterns. For example, the following standards are given for grade 5:

> *Generate a pattern using a written description.*
>
> *Identify, describe, and continue patterns presented in a variety of formats (e.g., numeric, visual, oral, written, kinesthetic, pictorial).*
>
> *Recognize and create patterns of change from everyday life using numerical or pictorial representations.*
>
> *Generalize patterns of change and recognize the same general patterns presented in different representations.*
>
> *Use probability to generalize from a simple pattern or set of examples and justify why the generalization is reasonable.*

Standards on patterns are also featured in the middle school standards. The study of patterns appears to be an end in itself in the New Mexico standards, with little connection to middle school mathematics. And finally, the standards for grades 9-12 make no mention of completing the square or proving the Pythagorean Theorem.

Some shortcomings of these standards could be corrected with systematic editing by someone knowledgeable in mathematics. For example, one of the grade 9-12 standards is:

> *Work with composition of functions (e. g., find f of g when* f(x) = 2x - 3 *and* g(x) = 3x-2)*, and find the domain, range, intercepts, zeros, and local maxima or minima of the final function.*

This standard asks students to find maxima and minima only for compositions of functions. There is no hint of calculus in any of the grade 9-12 standards, but even if calculus was developed in these grades, why restrict the identification of local extreme values only to compositions of functions? In the particular example listed above, since the two functions f(x) and g(x) are linear, their composition is also linear, and there are no maximum or minimum values of the composition.

Other Problems

A few general criticisms are in order. The New Mexico standards document announces on page three that "Electronic technologies such as calculators and computers are essential tools for teaching, learning, and doing mathematics," a clear example of false doctrine. (That being said, the rest of New Mexico's standards do not actually overemphasize calculator use.) The glossary should be edited and improved. There is a general overemphasis on probability and statistics throughout; probability is introduced prematurely in Kindergarten and continues through the lower grades, before students have mastered the facility with fractions needed to understand the topic. Prerequisites are not developed for the seventh-grade standard, "Approximate a line of best fit for a data set in a scatter plot form and make predictions using the simple equation of that line." To do this properly is college-level mathematics, and to do it any other way is not mathematics.

New York

Reviewed: *Mathematics Resource Guide with Core Curriculum,* 1999. The *Core Curriculum* is organized in two-year grade bands: PK-K, 1-2, 3-4, 5-6, 7-8, and, for high school, Math A and the more advanced Math B curriculum. Performance Indicators are categorized according to seven strands or "Key Ideas": Mathematical Reasoning; Number and Numeration; Operations; Modeling/Multiple Representation; Measurement; Uncertainty; and Patterns/Functions. The performance indicators are followed by sample classroom lessons. Assessment examples are also provided for grades 4, 8, and for Math A.

The New York State standards were in revision in 2004, but at the time of this writing the committee had not produced a final draft. Accordingly we reviewed the 1999 document that was also the basis of the 2000 Fordham review. Our scores are lower than the scores in the previous review largely because of our less optimistic interpretation of the many ambiguities in New York's standards.

2005 STATE REPORT CARD

New York

Clarity: 1.50	D
Content: 2.33	C
Reason: 2.00	C
Negative Qualities: 2.25	C
Weighted Score: 2.08	Final Grade: C

2000 Grade: **B**

1998 Grade: **B**

The classification scheme of the performance indicators, according to the Key Ideas, compromises the quality of New York's standards. Too much emphasis is placed on patterns, probability, and data analysis. The sixth Key Idea, "Uncertainty," which includes performance indicators for estimation and probability, is explained as:

> *Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.*

This "Key Idea" is misleading and a poor choice of category for performance indicators. It mistakenly associates ambiguities inherent in choosing mathematical models for "everyday situations" with mathematics itself.

Wastin' Time

The sample classroom lessons are often little more than puzzles and are poor vehicles for teaching core principles of mathematics. They can be enormous time-wasters too. For example, one grade 5-6 classroom idea is:

> *Students use the library to research kite history and learn to identify various kinds of kites. They design a*

particular kind of kite (of geometric shape), construct it, decorate it, and fly it in a contest. . . .

Many performance indicators ambiguously direct students to "explore," "relate," "consider," or "investigate." For example, in grades 3 and 4, students are asked to "consider, discuss, and predict whether the sum, difference, or product of two numbers is odd or even" and to "use counters to explore number patterns like triangular numbers and square numbers." It is unclear from these indicators what students are expected to know. Some of the performance indicators are unclear in other ways, such as this directive for fifth-and sixth-grade students: "Have an understanding of the basic characteristics of a variable," or the Math B indicator: "Use slope and midpoint to demonstrate transformations."

The *Core Curriculum* prematurely introduces calculators in grades 1-2. Fraction calculators are recommended for the intermediate grades, and scientific and graphing calculators are recommended for high school. The use of calculators in the early grades, and fraction calculators in the middle grades, compromises standards addressing rational number arithmetic, and even at the high school level calculators undermine basic graphing and algebra skills. For example, a recommended activity for Math B is:

Use your graphing calculator to graph $y = x^2 - 1$. *Compare the x values of where the graph crosses the axis and the solution to the equation* $x^2 - 1 = 0$.

Students should be able to find the graph of $y = x^2 - 1$ without calculator assistance. Completing the square is a powerful and important technique for graphing conic sections, including parabolas, and for deriving the quadratic formula, but nowhere in New York's Math A and Math B standards are students explicitly asked to complete the square of a quadratic polynomial.

A Credible Course

In spite of the negative role played by calculators, taken as a whole the Math A and Math B Performance Indicators outline a credible course of study for high school students. However, there is too much redundancy in the performance indicators for middle and high school grades. For example, right triangle trigonometry requirements appear repeatedly:

Grades 5-6

Develop readiness for basic concepts of right triangle trigonometry.

Grades 7-8

Find the measure of the sides and angles of a right triangle, using the Pythagorean Theorem and trigonometric ratios.

Explore and develop basic concepts of right triangle trigonometry.

Develop and apply the formulas for sine, cosine, and tangent ratios.

Math A

Use trigonometry as a method to measure indirectly.

- *Right triangle trigonometry.*

Math B

Use trigonometry as a method to measure indirectly.

- *Triangle solutions.*
- *Right triangle trigonometry.*

The emphasis on trigonometry in grades 5-8 is misplaced considering the weak development of algebra by the end of eighth grade and the continuing attention to fraction arithmetic. Division of fractions is also poorly presented in this Performance Indicator:

Demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc.).

- *Divide fractions, using a variety of approaches: factor product, partitioning, measurement, common denominator, and multiply by the reciprocal.*

It is unclear what methods are intended by "factor product, partitioning, measurement, common denominator," but it is essential that students be presented with a clear definition of the meaning of fraction division.

A positive feature of the elementary grade standards is that students are required to memorize the basic number facts. However, no mention is made of the standard algorithms of arithmetic. Whole number computations are expected, but students evidently invent their own algorithms, as indicated, for example, in the following Performance Indicator: "Develop strategies for selecting the appropriate computational and operational method in problem-solving situations." Probability is introduced far too early. For first and second grade, students are expected to "predict experimental probabilities" long before they have a firm grasp of fractions. The focus on data collection is obsessive and strays too far away from mathematics in the direction of social science, as in this grade 3-4 performance indicator:

> *Make predictions, using unbiased random samples.*
>
> - *Collect statistical data from newspapers, magazines, polls.*
> - *Use spinners, drawing colored blocks from a bag, etc.*
> - *Explore informally the conditions that must be checked in order to achieve an unbiased random sample (i.e., a set in which every member has an equal chance of being chosen) in data gathering and its practical use in television ratings, opinion polls, and marketing surveys.*

North Carolina

Reviewed: *Mathematics: Standard Course of Study and Grade Level Competencies*, revised 2003. The document provides grade-level standards for each of the grades K-12, as well as 12 individual courses in high school.

2005 STATE REPORT CARD

North Carolina

Clarity: 2.33	C
Content: 1.50	D
Reason: 1.50	D
Negative Qualities: 2.25	C
Weighted Score: 1.82	Final Grade: C
2000 Grade: A	
1998 Grade: A	

North Carolina's recent revision of its standards has proved to be a misstep. Though these standards are reasonably clear, content coverage is mediocre at all levels, with pervasive shortcomings such as an overemphasis on patterns, data analysis, and probability, and an inappropriate use of technology.

Students are encouraged to use "appropriate technology" starting in second grade, when they "will solve relevant and authentic problems using appropriate technology." In grades 3-5, as part of their algebra instruction, students

> *. . . continue to identify and describe patterns in many situations. Tools, such as calculators and computers, are used to investigate and discover patterns.*

The educational goal of discovering patterns is not made clear. Similarly, third-graders are urged to "develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil." The decision whether to use calculators or not appears to be at least partly left to the students, which can be a potential roadblock to learning arithmetic.

The arithmetic of whole numbers is poorly developed in elementary school. Memorization of the single-digit number facts is not explicitly required, nor is familiarity with the standard algorithms of arithmetic. In second grade, students "develop fluency with multi-digit addition and subtraction through 999 using multiple strategies"—but which strategies is left unclear.

Missing Content

Long division is never mentioned in these standards and, while place value is mentioned, there is no reference to carrying or borrowing. Fraction arithmetic is insufficiently developed. For example, in fifth grade, students are asked to:

> *Develop fluency in adding and subtracting non-negative rational numbers (halves, fourths, eighths; thirds, sixths, twelfths; fifths, tenths, hundredths, thousandths; mixed numbers).*

a) Develop and analyze strategies for adding and subtracting numbers.

b) Estimate sums and differences.

c) Judge the reasonableness of solutions.

Fifth-grade students are evidently not expected to be able to add sevenths or elevenths to other fractions, for no apparent reason. The standards devoted to the arithmetic of fractions give scant attention to the reasoning behind fraction arithmetic, including the meaning of division.

As part of the third-grade data analysis and probability standards, students are expected to "collect, organize, analyze, and display data (including circle graphs and tables) to solve problems." But the skills required to construct circle graphs in a mathematically meaningful way (such as understanding angles and practice with a protractor) are not established by third grade. Indeed, the word "angle" does not even appear until fifth grade in these standards.

Surprisingly, "mean" (or average) is not introduced until grade 7, in spite of the heavy emphasis on data analysis and probability throughout the document.

Problems in High School

At the high school level, in the "Introductory Mathematics" standards, the focus within the Number and Operations strand is on irrational numbers. The authors appear to take the view that since arithmetic of rational numbers is covered by grade 7, the next topic to emphasize is irrational and real numbers. The standard, "develop number sense for the real numbers," is followed by "define and use irrational numbers." But for what purpose are students expected to use irrational numbers? This latter directive is vague. Do the authors have in mind working with and simplifying numerical expressions that include multiples of pi, square roots of nonperfect squares, and so forth? Specific arithmetical computations can be carried out only for special examples of irrational numbers.

The Algebra I course is weak, with an emphasis on manipulatives, calculators, and software that is inappropriate and potentially counterproductive. Completing the square and the quadratic formula are not mentioned in any of the high school standards, including Algebra I and II. There is little if any attention given to mathematical reasoning in these courses. The inclusion of standards devoted to the use of matrices to display and interpret data in the Algebra I course is out of place and there is no substitute for missing topics, such as simplifying, adding, multiplying, and dividing rational functions–topics that are also missing in the Algebra II standards. In Algebra II, coverage of conic sections is limited to circles and parabolas. "Interpret the constants and coefficients" is an aimless directive given for quadratic and cubic polynomials, rational functions, inequalities with absolute value, and other categories.

Prior to ninth grade, the standards include no mention of any specific class of triangles: right, isosceles, equilateral, etc. The term "right triangle" appears only in the standards for grades 9-12. The terms "acute angle," "obtuse angle," "complementary angle," "supplementary angle," "vertical angle," and "adjacent angle" do not appear anywhere in this 81-page document.

The focus on manipulatives and technology in grades 9-12 is excessive, as in these standards:

Students use technology to assist in developing models and analytical solutions.

Appropriate technology, from manipulatives to calculators, should be used regularly for instruction and assessment.

No guidance is provided, however, for the use of manipulatives, graphing calculators, and computers.

Finally, the standards for Advanced Placement Calculus are poorly written, redundant, and in some cases almost incomprehensible, as in these two examples:

Demonstrate an understanding of limits both local and global.

Recognize and describe the nature of aberrant behavior caused by asymptotes and unboundedness.

North Dakota

Reviewed: *North Dakota Mathematics Standards and Benchmarks: Content Standards–Draft*, dated January 2004, was approved by the North Dakota Department of Public Instruction on February 3, 2004. This document provides standards for each of the grades K-8, plus one set of standards for grades 9 and 10 and another for grades 11 and 12.

2005 STATE REPORT CARD

North Dakota

Clarity: 2.33	**C**
Content: 1.33	**D**
Reason: 1.00	**D**
Negative Qualities: 3.00	**B**
Weighted Score: 1.80	Final Grade: C
2000 Grade: **D**	
1998 Grade: **D**	

The North Dakota standards are straightforward and are presented in an easily readable format. They are arranged in five strands, which focus on arithmetic, geometry, algebra, measurement, and data. Each strand has subheadings with descriptive titles (e.g., "Coordinate Geometry" and "Probability"). Unlike other states, North Dakota sensibly avoids creating standards for every topic in every grade. For example, under the Probability Strand for Kindergarten and first grade, the standards simply state, "No expectations at this level." Other states would do well to emulate this feature.

The development of arithmetic in elementary school is strong. Students are required to memorize basic number facts at the appropriate point and to calculate with whole numbers, fractions, and decimals, as in standards such as, "Divide multi-digit numbers by a single-digit number," and "Add and subtract improper fractions and mixed numbers with unlike denominators."

Lack of Coherence

However, the elementary standards have some shortcomings. The number line, which should be introduced in Kindergarten, does not appear until fourth grade. Some of the computational standards should include the phrase "using the standard algorithm." Calculators are introduced in third grade with the standard,

> *Use a variety of methods and tools for problem solving; e. g., computing, including mental math, paper and pencil, calculator, manipulatives.*

Guidance on calculator use is not provided in this document. With widely differing views on what constitutes appropriate calculator use among teachers and administrators, the introduction of calculators in third grade has the potential to undermine otherwise credible arithmetic standards. Also, North Dakota's standards would benefit from the inclusion of standards that explicitly call for solutions to multi-step word problems that combine several operations.

There is a lack of coordination between the developments of fraction and decimal arithmetic. Fifth-grade students are expected to multiply and divide multi-digit decimals, but the concept of multiplication and division of fractions is not introduced until sixth grade.

The measurement standards are direct and grade-appropriate, as in the fifth-grade standard, "measure angles using protractors." But some standards should be more specific, and the development of some topics is too slow. The sixth grade, "convert unit measurements within the same system (metric and standard)," for example, should specify which conversions students should be able to do. Students measure length to the nearest inch in second grade, nearest half-inch in third grade, nearest quarter-inch in fourth grade, nearest eighth-inch in fifth grader and nearest sixteenth-inch in sixth grade. Does this skill really require five years to develop? Only in eighth grade are students expected to know that a yard is roughly the same as a meter.

Area is introduced in third grade, when students

> *estimate and measure perimeter, area, and volume using links, tiles, grid paper, geoboards, and dot paper.*

Then in fifth grade, students are explicitly required to "use formulas to calculate the perimeter and area of squares and rectangles." In sixth grade they are explicitly called upon to find the area of a triangle. But nowhere in the North Dakota standards are students required to understand how to derive or deduce any formula for area, perimeter, or volume of any geometric figure or solid. Instead, they are asked only to *use* formulas.

Probability and statistics are overemphasized, but the K-6 geometry and algebra strands are generally solid. Nevertheless, a few are below grade level, such as the sixth grade standard, "Identify polygons; i.e. triangle, rectangle, square, rhombus, parallelogram, trapezoid, pentagon, hexagon, octagon." Several standards leave one wondering what is intended. For example:

> *Use parentheses in solving simple equations.*

> *Use equations to solve problems (e.g. 28/x=7).*

Many grade 7-12 standards fall below grade level relative to other states. Rates do not appear until eighth grade, and students are not required to graph and solve general linear equations until tenth grade.

Letdown in the Later Years

The high school algebra standards are weak and often vague. What does it take to "draw conclusions about a situation being modeled"? The geometry standards require little reasoning and also suffer from a lack of specificity. What is meant by "represent shapes using coordinate geometry"? And what will teachers and students make of the following highly inflated standard?

> *Use geometric models to gain insights into, and answer questions in, other areas of mathematics, other disciplines, and other areas of interest; e.g., art and architecture.*

Only a single standard mentions proofs, and it gives no hint as to what students are expected to be able to prove. There is an evident lack of coherence in the grade 7-12 standards, and they do not outline a program of study that adequately prepares students for college.

Ohio

Reviewed: *Academic Content Standards*, December 11, 2001. This 240-page framework includes specific grade-level indicators for grades K-12.

2005 STATE REPORT CARD

Ohio

Clarity: 2.00	C
Content: 1.33	D
Reason: 1.00	D
Negative Qualities: 1.50	D
Weighted Score: 1.43	Final Grade: D

2000 Grade: **A**

1998 Grade: **A**

Ohio's 2001 revision of its math standards turned out to be a dreadful mistake. There are serious deficiencies in these standards, including coverage of arithmetic and the algebra indicators. Completing the square is absent and consequently there is no expectation that students should understand the derivation of the quadratic formula. There is too much emphasis on the study of patterns as an end in itself, with little connection to mathematics. Statistics and probability are grossly overemphasized throughout and sometimes require mathematics not yet covered in the other strands. The glossary would benefit from editing. While the geometry strand, especially for high school, is nicely developed, it does contain an egregious example of false doctrine in one of the sixth-grade indicators:

> *Draw circles, and identify and determine relationships among the radius, diameter, center and circumference; e.g., radius is half the diameter, the ratio of the circumference of a circle to its diameter is an approximation of* π.

This last point is simply wrong; the number π is exactly the ratio of the circumference of a circle to its diameter, not merely an approximation. In addition, geometry contains an example of significant inflation in one of the grade 12 indicators:

> *Recognize and compare specific shapes and properties in multiple geometries; e.g., plane, spherical, and hyperbolic.*

The Ohio framework document lists as a guiding principle to "incorporate use of technology by ALL students in learning mathematics." The emphasis on technology as an end in itself is one of the defects of this document, in some cases working against mathematical reasoning, as in this grade 7 indicator:

> *Describe differences between rational and irrational numbers; e.g., use technology to show that some numbers (rational) can be expressed as terminating or repeating decimals and others (irrational) as non-terminating and non-repeating decimals.*

The technology is not specified, but calculators cannot establish the fact that rational numbers necessarily have repeating decimals. Indeed, technology poses a barrier to this understanding. The long division algorithm, in contrast, serves this purpose well. However, it's mentioned nowhere in the Ohio document. This indicator contributes to false doctrine, and detracts from reasoning and content.

Arithmetic Problems

Most egregious, though, are problems with the fundamental arithmetic strand. Standards for the elementary school grades call for "fluency" with the single-digit number facts, but they do not explicitly call for memorization. None of the grade-level indicators requires students to learn the standard algorithms of arithmetic. The only occurrence of the term "standard algorithm" is in this benchmark for grades 5-7:

> *Use and analyze the steps in standard and non-standard algorithms for computing with fractions, decimals, and integers.*

The indicators for fractions are poorly developed and undermine the use of clear definitions and mathematical reasoning. For example, one of the fifth grade indicators is:

> *Use various forms of "one" to demonstrate the equivalence of fractions; e.g.,* $^{18}/_{24} = {}^{9}/_{12} \text{ x } {}^{2}/_{2} = {}^{3}/_{4} \text{ x } {}^{6}/_{6}$.

Defining equivalence of fractions via multiplication of fractions is circular and confusing. The concept of equivalence of fractions is fundamental to the arithmetic of fractions and must be clearly developed before the arithmetic operations for fractions can even be defined. This can be achieved by defining two fractions to be equivalent if they represent the same point on a number line, and therefore the same number. It follows that multiplying both the numerator and denominator of a fraction by the same counting number results in an equivalent fraction, and the cross multiplication criterion for equivalent fractions then also follows. The concept of equivalent fractions should make no reference to fraction multiplication (only to whole number multiplication).

Division of fractions first appears for grade 6:

> *Represent multiplication and division situations involving fractions and decimals with models and visual representations; e.g., show with pattern blocks what it means to take* $2^{2}/_{3} \div {}^{1}/_{6}$.

Regrettably, this indicator misrepresents the meaning of division of fractions, which is not accomplished by successive subtractions, except in the rare cases where the quotient is an integer, and therefore cannot be defined as repeated subtraction. Fraction division must be defined as the inverse operation to multiplication. For example, 1/4 divided by 1/3 cannot be understood as repeated subtraction, since 1/3 is greater than 1/4 and therefore cannot be subtracted from 1/4 even once. The requirement that sixth-graders use manipulatives to carry out fraction division, even aside from the misleading nature of this grade-level indicator, is an example of the weak treatment of fractions in the Ohio standards.

The other arithmetic operations are also handled inadequately, as in this sixth-grade indicator:

> *Develop and analyze algorithms for computing with fractions and decimals, and demonstrate fluency in their use.*

While it is essential that students learn to calculate symbolically with paper and pencil, this standard glosses over the crucial point that the arithmetic operations for fractions must be clearly defined. Practice in calculations using the definitions is essential before students try to develop algorithms on their own, a questionable activity in the case of fraction arithmetic.

Oklahoma

Reviewed: *Priority Academic Student Skills: Mathematics Content Standards*, August 22, 2002. Standards are provided for each of the grades 1-8, along with course standards for Algebra I, Geometry, and Algebra II.

2005 STATE REPORT CARD

Oklahoma

Clarity: 2.17	C
Content: 1.83	C
Reason: 1.50	D
Negative Qualities: 2.50	B
Weighted Score: 1.97	Final Grade: C

2000 Grade: **B**

1998 Grade: **F**

Oklahoma's mediocre standards, revised since our last review, fall into nearly all of the traps we see again and again in math standards. The standards for grades 1-5 do not require memorization of the basic number facts; instead, they simply call upon students to "demonstrate fluency" with them. Might "fluency" mean, for example, the rapid use of finger counting to deduce the basic facts? If memorization is expected, the Oklahoma document should make that clear. The failure to require any use or understanding of the standard arithmetic algorithms is another significant shortcoming. Rather than familiarizing students with these efficient algorithms and the reasoning behind them, the Oklahoma standards call for students to invent algorithms, as illustrated by this fifth-grade standard: "Develop division algorithms (e.g., use physical materials to show 12 objects arranged in 3 groups, show division as repeated subtraction and as the inverse of multiplication)."

Potentially undermining mathematical reasoning, fifth-grade students are expected to multiply and divide decimals despite the fact that multiplication and division of fractions are not introduced until sixth grade. A realistic possibility then exists that instruction consistent with the Oklahoma standards might introduce multiplication of decimals as a rote process with little or no meaning attached to it.

Too Much Technology

Calculators are inappropriately recommended (under the heading "Suggested Materials") for grades 1-5. The Overview of those standards does include the admirable reminder, "Calculators do not replace the need for students to be fluent with basic facts, have efficient computation strategies, be able to compute mentally, and do paper-and-pencil computation." Nevertheless, the inclusion of calculators in all of the elementary grades without guidance or justification undermines arithmetic instruction for Oklahoma students.

Technology and manipulatives are overemphasized in several parts of the document. For example, the "Fifth Grade Suggested Materials Kit" includes snap cubes, rods, one-inch color tiles, calculators, boxes, pawns, number cubes, balance scale, fraction strips, tangrams, spinners, base-10 blocks, pattern blocks, fraction and decimal towers, geoboards, and computer tessellation software. A similar list is recommended for the middle grades. The introduction to each of the course standards for Algebra I, Geometry, and Algebra II includes the statement, "Visual and physical models, calculators, and other technologies are recommended when appropriate and can enhance both instruction and assessment." The overuse of manipulatives, particularly in the middle and upper grades, works against sound mathematical content and instruction, and undermines the abstract nature of mathematics itself. The recommendation that physical models be included with assessments at the high school level is absurd.

The middle school standards require calculations with rational numbers, but they provide few details. Exponents are developed through the use of patterns, but little attention is given to definitions or to mathematical reasoning. No standard addresses the topic of irrational numbers, except perhaps implicitly in the vague sixth-grade standard, "Convert, compare and order decimals (terminating and nonterminating), fractions, and percents using a variety of methods."

The high school course standards develop a broad range of topics, including complex numbers, conic sections, exponential and logarithmic functions, finding roots of polynomials and rational functions, asymptotes of rational functions, as well as a variety of topics in geometry. Some attention is given to trigonometry within the geometry standards, but not enough. The Algebra I and II course standards inappropriately include statistics standards and call upon students to find lines of best fit for data.

Another misplaced standard listed under Algebra II is:

> *Graph a polynomial and identify the x- and y-intercepts, relative maximums and relative minimums.*

Identifying relative maximum and minimum values of polynomials (except for quadratic polynomials) is a calculus topic, not an Algebra II topic.

Oregon

Reviewed: *Mathematics Grade-level Standards & K-2 Foundations*, April 2002; *Content Standards; Newspaper: Grade Level Foundations and Standards,* 2003-2004 School Year. Oregon provides grade-level standards for each of the grades K-8 and standards for the "Certificate of Initial Mastery" (CIM). The CIM standards apply to the band of grades 9-12. For the high school grades, there is also a "Certificate of Advanced Mastery," but we did not evaluate those standards. The standards evaluated here, for which students will first be accountable on the 2004-05 state assessments, are organized by the strands: Calculations and Estimation; Measurement; Statistics and Probability; Algebraic Relationships; Geometry; and Mathematical Problem Solving.

2005 STATE REPORT CARD

Oregon

Clarity: 2.50	**B**
Content: 1.00	**D**
Reason: 0.00	**F**
Negative Qualities: 2.25	**C**
Weighted Score: 1.35	Final Grade: **D**

2000 Grade: **D**

1998 Grade: **D**

Oregon's revised standards remain woefully incomplete. Some of the Calculation and Estimation standards are reasonable and appropriate, including those that address use of the number line and the identification and comparison of integers. Decimals, fractions, and percents are introduced at appropriate grade levels. The geometry and measurement strands also treat topics well, in general. However, the standards devoted to arithmetic have serious shortcomings. Students are not explicitly required to memorize the basic number facts. Instead, in third grade, for example, students are to "develop and acquire efficient strategies for determining multiplication and division facts 0-9." In fourth grade, students "apply with fluency efficient strategies for determining multiplication and division facts 0-9."

Using efficient strategies to deduce the fact that *6 x 7 = 42* is not the same as memorizing that fact. The Oregon standards for the elementary grades do require students to perform calculations for whole numbers, but students are not required to use the standard algorithms of arithmetic. Instead, fourth-graders "develop and evaluate strategies for multiplying and dividing whole numbers." Developing and evaluating strategies is not the same as understanding and being able to use the conventional algorithms of arithmetic.

The standards for fraction arithmetic lack coherence. The first explicit reference to equivalent fractions appears in sixth grade in the standard, "Apply factors and multiples to express fractions in lowest terms and identify fraction equivalents." Yet, without the fundamental notion of equivalent fractions, a necessary prerequisite to fraction arithmetic, fourth-graders are expected to, "Add and subtract commonly used fractions with like denominators (halves, thirds, fourths, eighths, tenths) and decimals to hundredths." Fifth-graders are expected to compute with decimals, as indicated by this fifth-grade standard: "Add, subtract, multiply, and divide decimals, including money amounts." Computational procedures are evidently left to the students as indicated in this fifth-grade standard: "Develop and evaluate strategies for computing with decimals and fractions." Exhortations to develop unidentified algorithms for computation continue to 8th grade: "Develop and analyze algorithms and compute with rational numbers."

The development of algebra in the middle grades and high school standards is weak and the pace slow. Probability and statistics are overemphasized, not only in the high school grades, but throughout.

Many standards at all grade levels are poorly written, vague, or inflationary, such as these:

> *Identify and describe situations with constant or varying rates of change and compare them.*
>
> *Model and solve contextualized problems using various representations such as graphs, tables, and equations.*
>
> *On a coordinate plane, determine the relative placement of two lines.*
>
> *Determine and interpret maxima or minima and zeros of quadratic functions, and linear functions where y = constant.*
>
> *Determine a shape that has minimum or maximum perimeter, area, surface area, or volume under specified conditions.*
>
> *Accurately solve problems using mathematics.*
>
> *Recognize that taking the nth root of a number corresponds to prime factorization.*

This last standard is particularly egregious because it asks students to recognize as true something that is completely false.

Pennsylvania

Reviewed: *Academic Standards for Mathematics*, 2004, which consists of standards for grades 3, 5, 8, and 11.

2005 STATE REPORT CARD

Pennsylvania

Clarity: 1.33	D
Content: 1.17	D
Reason: 1.00	D
Negative Qualities: 1.75	C
Weighted Score: 1.28	Final Grade: D
2000 Grade: **C**	
1998 Grade: **D**	

Pennsylvania's recent revision of its math standards was a step backward for a state that has struggled to develop a solid set of mathematics expectations for students. In the elementary grades, the new framework is ambiguous as to whether students should memorize the addition, subtraction, multiplication, and division facts, instead asking students to "demonstrate knowledge of basic facts in four basic operations." More promising are third-grade standards that address the algorithms of addition and subtraction: "Solve single- and double-digit addition and subtraction problems with regrouping in vertical form" and "Explain addition and subtraction algorithms with regrouping." A similar fifth-grade standard calls for understanding of unspecified multiplication and division algorithms. The requirement that students understand the algorithms they use is commendable. However, it is not clear that students are expected to learn the conventional arithmetic algorithms. Indeed, one fifth-grade standard calls for students to develop their own algorithms: "Develop and apply algorithms to solve word problems that involve addition, subtraction, and/or multiplication with decimals with and without regrouping."

Fraction arithmetic is developed poorly, with too much reliance on models and calculators. At the fifth grade level, students are asked to:

> *Use models to represent fractions and decimals.*
>
> *Develop and apply algorithms to solve word problems that involve addition, subtraction, and/or multiplication with fractions and mixed numbers that include like and unlike denominators.*
>
> *Demonstrate skills for using fraction calculators to verify conjectures, confirm computations and explore complex problem-solving situations.*

With the exception of one probability standard, these are the only fifth-grade standards that address fractions.

An egregious example of false doctrine appears in the geometry standards for fifth grade: "Describe the relationship between the perimeter and area of triangles, quadrilaterals and circles." This standard suggests that area and perimeter are related for triangles and quadrilaterals, but there is actually no functional relationship between area and perimeter.

Vague Content

As in many states, probability and statistics are overemphasized throughout the grades. Third-graders are prematurely expected to "predict and measure the likelihood of events and recognize that the results of an experiment may not match predicted outcomes." In grades 8 and 11, the probability and statistics standards stray too far from mathematics and too close to social science, as in these standards:

> *Determine the validity of the sampling method described in studies published in local or national newspapers.*
>
> *Analyze predictions (e.g., election polls).*
>
> *Use appropriate technology to organize and analyze data taken from the local community.*
>
> *Describe questions of experimental design, control groups, treatment groups, cluster sampling and reliability.*

An eleventh-grade standard calls for students to "describe and use" the normal distribution, even though its understanding requires calculus, a topic only poorly covered in these standards. The framework absurdly lists "Concepts of Calculus" standards at all four grade levels. For example, fifth-graders are asked to "identify maximum and minimum." This directive is given without specifying the type of quantity for which the extremes are to be found. Even the eleventh-grade standards in this category have little substance. Without any mention of limits, derivatives, or integrals, and no further elaboration, the eleventh-grade standards call for students to "determine maximum and minimum values of a function over a specified interval" and "graph and interpret rates of growth/decay." Without the prerequisite skills, which are not covered in these standards, these are impossible tasks except at the level of pressing buttons on a calculator.

The Pythagorean Theorem is mentioned just once in eighth grade and once in eleventh grade, in both cases only in the context of problem-solving. Students are not expected to see a proof of the theorem. The algebra standards are weak and certainly would not support credible calculus standards. For example, the only standard that addresses the roots of quadratic polynomials is: "Solve linear, quadratic and exponential equations both symbolically and graphically."

The trigonometry standards are also weak, and do little more than treat trigonometric functions as images on the screens of graphic calculators ("Use graphing calculators to display periodic and circular functions; describe properties of the graphs.") By the eleventh grade, graphing calculators and computer software become ends in themselves, with students being asked to "demonstrate skills for using computer spreadsheets and scientific and graphing calculators."

Rhode Island

Reviewed: *Mathematical Power for ALL Students: The Rhode Island Mathematics Framework K-12*, 1995 includes standards for the grade bands K-4, 5-8, 9-10, and 11-12. *Draft K-8 New Hampshire and Rhode Island Local and NECAP Grade Level Expectations* (GLEs), June 6, 2004.

2005 STATE REPORT CARD

Rhode Island

Clarity: 1.00	**D**	
Content: 0.67	**F**	
Reason: 0.00	**F**	
Negative Qualities: 1.00	**D**	
Weighted Score: 0.67	Final Grade:	F
2000 Grade: **F**		
1998 Grade:**F**		

This year, Rhode Island and New Hampshire jointly implemented grade-level expectations associated with the New England Common Assessment Program used in both states (plus Vermont) in grades 3-8. They fail to budge Rhode Island's failing grade.

Some of the content standards in the *Framework* are so vague that it is difficult to discern their meaning. For grades K-4 one finds, "Through problem-solving situations, all students will construct their own understanding, so that by the end of fourth grade they will: . . . Use patterns to communicate relations." What does this mean? No explanation is provided. For middle school, the standards include, "Through problem-solving situations, all students will construct their own understanding, so that by the end of eighth grade they will: . . . Investigate inequalities." Again, completely unclear. Similarly for grades 11-12, the Rhode Island Math *Framework* gives as a standard, "Through problem-solving situations, all students will construct their own understanding, so that by the end of twelfth grade they will: . . . Investigate and compare various geometries." Does this last standard require eleventh-or twelfth-grade students to study non-Euclidean geometries and to compare them? Given the weakness of the geometry standards for plane Euclidean geometry in the Rhode Island *Framework*, such a standard is unrealistic.

Other examples of vague standards for grades 11 and 12 are: "Select and apply trigonometric functions to solve problems" and "Deduce properties of, and relationships between figures, given assumptions," with no further elaboration. The range of possible interpretations of these standards is so broad that it renders them effectively meaningless.

Missing Fundamentals

Arithmetic is insufficiently developed in the *Framework*. Students are not directed to memorize the basic number facts or to master the standard algorithms of arithmetic.

Algebra is absent from these standards, except superficially. In grades 9 and 10, where algebra standards should appear prominently, one finds, "Through problem solving situations, all students will construct their own understanding, so that by the end of tenth grade they will: . . . Have an intuitive understanding of algebraic procedures." The terms "polynomial," "quadratic formula," and "Pythagorean Theorem" do not appear in the Rhode Island standards. Standards under the heading "Patterns, Relations, and Algebra" require investigations of patterns with no clear goals; they do not present a systematic development of algebra.

The geometry standards are similarly poorly developed. In fourth grade, students are called upon to describe "shapes," and in higher grades, "figures," but there is no mention of specific shapes or figures. Indeed, the word "triangle" does not even appear in these standards.

The *Framework* overemphasizes the importance of technology at all grade levels, and discourages the use of textbooks, as in this passage:

> *Traditionally, the mathematics textbook has dictated the mathematics curriculum in most schools. The intent of this section is to address the need to shift from using one resource, a textbook, to using multiple materials and resources.*

Much can be said in favor of a well-written math textbook, but the Rhode Island *Framework* does not recognize any such positive features. The mathematics program advocated for Rhode Island students in this *Framework* is one of unending brainstorming and student-discovery, with assistance from the Internet and technology. Lacking is a coherent development of K-12 mathematics and recognition of any hierarchy of prerequisites necessary to achieve a sound mathematical education.

GLEs: No Improvement

The GLEs make no reference to calculators, but like the *Framework*, they overemphasize manipulatives. Students are not required to memorize the basic number facts or to use or understand the standard algorithms of arithmetic.

The GLEs frequently suffer from convoluted writing, as illustrated by this fifth-grade standard:

> *M5:1 Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from 0 to 9,999,999 through equivalency, composition, decomposition, or place value using models, explanations, or other representations; positive fractional numbers (proper, mixed number, and improper) (halves, fourths, eighths, thirds, sixths, twelfths, fifths, or powers of ten [10, 100, 1000]), decimals (to thousandths), or benchmark percents (10%, 25%, 50%, 75%, or 100%) as a part to whole relationship in area, set, or linear models using models, explanations, or other representations.**

The asterisk in the last line references a footnote that places confusing restrictions on the rational numbers that students consider:

> **Specifications for area, set, and linear models for grades 5 - 8: Fractions: The number of parts in the whole are equal to the denominator, a multiple of the denominator, or a factor of the denominator. Percents: The number of parts in the whole is equal to 100, a multiple of 100, or a factor of 100 (for grade 5); the number of parts in the whole is a multiple or a factor of the numeric value representing the whole (for grades 6-8). Decimals (including powers of ten): The number of parts in the whole is equal to the denominator of the fractional equivalent of the decimal, a multiple of the denominator of the fractional equivalent of the decimal, or a factor of the denominator of the fractional equivalent of the decimal.*

Mathematical topics in the GLEs are poorly organized. In some cases the ordering of topics from one grade to the next makes no sense. Consider, for example, these fourth-and fifth-grade Expectations:

> *Demonstrates conceptual understanding of perimeter of polygons, and the area of rectangles, polygons, or irregular shapes on grids using a variety of models, manipulatives, or formulas. Expresses all measures using appropriate units.*

> *Demonstrates conceptual understanding of perimeter of polygons, and the area of rectangles or right triangles through models, manipulatives, or formulas, the area of polygons or irregular figures on grids, and volume of rectangular prisms (cubes) using a variety of models, manipulatives, or formulas. Expresses all measures using appropriate units.*

Fourth-graders "demonstrate conceptual understanding of . . . the area of polygons [sic]," while fifth-graders "demonstrate conceptual understanding" of right triangles. But triangles are polygons, and fourth-graders should understand how to find areas of rectangles and triangles before finding areas of more complicated polygons.

The only reference to slopes and linear functions in the GLEs for eighth grade is in this standard:

> *M(F&A)–8–2 Demonstrates conceptual understanding of linear relationships (y = kx; y = mx + b) as a constant rate of change by solving problems involving the relationship between slope and rate of change; informally and formally determining slopes and intercepts represented in graphs, tables, or problem situations; or describing the meaning of slope and intercept in context; and distinguishes between linear relationships (constant rates of change) and nonlinear relationships (varying rates of change) represented in tables, graphs, equations, or problem situations; or describes how change in the value of one*

variable relates to change in the value of a second variable in problem situations with constant and varying rates of change.

The emphasis on "varying rates of change" for nonlinear functions is misplaced. That subject is better treated in a calculus course. Eighth-graders could spend their time more profitably by learning that the slope *m* and y-intercept *b* in the equation $y = mx + b$ can be determined from the coordinates of any pair of points on its graph—one more example of the misguided approach of this document.

South Carolina

Reviewed: *Outlines of High School Mathematics Courses; Mathematics Course Standards*, 2000; *South Carolina Mathematics Curriculum Standards*, 2000. *The Curriculum Standards* provides standards for pre-K to 8, a single collection of standards for high school, and course standards for Algebra I, Algebra II, Geometry, Pre-calculus, and Probability and Statistics. These high school course standards consist mainly of the grade 9-12 standards from the *South Carolina Mathematics Curriculum Standards,* 2000, but they include some other standards as well. In addition to these grade and course standards, South Carolina provides course outlines for 13 high school courses, including those listed above, which identify suggested sequences of topics for instruction. School districts have the option of using the outlines.

2005 STATE REPORT CARD

South Carolina

Clarity: 1.00	**D**
Content: 1.67	**D**
Reason: 1.50	**D**
Negative Qualities: 0.75	**F**
Weighted Score: 1.32	Final Grade: **D**
2000 Grade: **B**	
1998 Grade: **D**	

The South Carolina standards, revised since our last review, excessively promote the use of technology in high school. The preface to the high school geometry standards begins, "The use of geometry software that supports a dynamic, interactive approach is essential to the instruction and assessment of geometry," an example of false doctrine. The use of geometry software is not essential to instruction and assessment of geometry. (If it were, most of the past 2000 years of geometry instruction and assessment would have been impossible.) The preface to the Algebra section of the grade 9-12 standards reads:

Hand-held graphing calculators are required as part of instruction and assessment. Students should use a variety of representations (concrete, numerical, algorithmic, graphical), tools (matrices, data), and technology to model mathematical situations in solving meaningful problems. Technology includes, but is not limited to, powerful and accessible hand-held calculators as well as computers with graphing capabilities.

The Algebra I standards also include the above paragraph, and electronic technologies are referenced in a number of algebra standards. For example:

Translate among and use algebraic, tabular, graphical, or verbal descriptions of linear functions using computer algebra systems, spreadsheets, and graphing calculators.

With and without using a graphing calculator, investigate, describe, and predict the effects of changing the slope and the y-intercept in applied situations.

Such standards do not contribute to sound instruction. Students will learn more by graphing linear equations by hand than from pressing buttons on graphing calculators, typing spreadsheets, or using computer algebra systems. Technology is also overblown in the measurement strand; the standards recommend the use of "calculator-based laboratories (CBLs), calculator-based rangers (CBRs), the Global Positioning System (GPS), digital micrometers, and infrared distance measurers."

In spite of the overemphasis on technology, the high school standards and course standards cover a broad range of topics, including trigonometry, the binomial theorem, a proof of the Pythagorean Theorem, completing the square of quadratic polynomials, and conic sections. However, the high school Algebra standards lack coherence. This Algebra II standard is strangely out of place: "Determine changes in slope relative to the changes in the independent variable." This directive is not supported by the necessary prerequisites. It belongs in a calculus course, not an algebra course.

Content Problems

Many standards are vague, inflated, or unclear, as these examples illustrate:

Connect geometry to other areas of mathematics, to other disciplines, and to the world outside the classroom. (Grade 4)

Describe, extend, and write rules for a wide variety of patterns. (Grade 6)

Draw a pair of perpendicular vectors to find a distance graphically. (Geometry)

Explain the use of a variable as a quantity that can change its value, as a quantity on which other values depend, and as generalization of patterns. (Grade 7)

Regarding this last example, beginning algebra should be understood as generalized arithmetic. At this level, a letter such as "x" is used to represent only a number and nothing more. Computation with an expression in x is then the same as ordinary calculations with concrete numbers. In this way, beginning algebra becomes a natural extension of arithmetic. It is misleading to give convoluted and esoteric explanations of "variable," much less to instruct students that a "variable" represents a "generalization of patterns."

As in many other states, probability and statistics are overemphasized throughout South Carolina's standards, as are patterns. An Algebra I standard calls upon students to "use patterns to generate the laws of exponents and apply them in problem-solving situations." Patterns can be used to suggest laws of exponents, but students should also justify laws of exponents by using fundamental properties of rational (or real) numbers, such as the commutative and associative properties of multiplication.

The elementary school standards require memorization of the basic number facts, including the multiplication and division facts by the end of third grade, a positive feature. Strangely, however, a fourth-grade standard appears a year after it is needed: "Recognize commutativity in the multiplication facts." Undermining the elementary school standards, calculators are introduced in second grade, and the standards make no mention of the standard algorithms of arithmetic.

South Dakota

Reviewed: *Mathematics Content Standards*, May 17, 2004. It provides standards for each of the grades K-8 as well as "core" high school standards and "advanced" high school standards. The standards are accompanied by examples and are organized into strands: Algebra, Geometry, Measurement, Number Sense, and Statistics and Probability. The document includes an elaborate set of performance descriptions for English Language Learners, at a lower level than performance standards for regular pupils.

2005 STATE REPORT CARD		
South Dakota		
Clarity: 2.17	C	
Content: 1.67	D	
Reason: 1.00	D	
Negative Qualities: 2.50	B	
Weighted Score: 1.80	Final Grade:	C
2000 Grade: **A**		
1998 Grade: **F**		

The 233-page document, which supplanted South Dakota's previous standards earlier this year, proved to be a misstep. The document is of uneven quality, with strong coverage of some areas undercut by weak development of some important skills.

A strong feature of the elementary grade standards is steady development of the algebra strand. Students gradually but systematically gain practice using the field properties to solve equations. There is, however, the usual tedious emphasis on patterns, including instances of false doctrine, such as in this fourth-grade standard: "Students are able to solve problems involving pattern identification and completion of patterns. Example: What are the next two numbers in the sequence? Sequence: 1, 3, 7, 13, __, __." Given only the first four terms of a pattern, there are infinitely many systematic, and even polynomial, ways to continue the pattern, and there are no possible incorrect fifth and sixth terms. Suggesting otherwise misleads students.

One third-grade standard requires students to "recall multiplication facts through the tens," but they are not explicitly called upon to memorize the basic addition, subtraction, or division facts. Whole number and decimal calculations are expected of students, but no mention is made of the standard algorithms of arithmetic, a significant shortcoming.

Slow Development of the Basics

The development of fractions proceeds slowly. The following sixth-grade standard is overly restrictive of the denominators of fractions that students are to consider: "Students are able to represent fractions in equivalent forms and convert between fractions, decimals, and percents using halves, fourths, tenths, hundredths."

The coordination of the development of fractions and decimals is also problematic. The standards call for the full development of decimal arithmetic by sixth grade, as shown by this standard: "Students are able to add, subtract, multiply, and divide decimals." However, the arithmetic operations for fractions are not developed until seventh grade: "Students are able to add, subtract, multiply, and divide integers and positive fractions."

The middle grade standards inappropriately restrict the types of algebra and geometry problems that students are expected to solve. For example, these seventh-and eighth-grade standards unnecessarily exclude fraction values:

> *Write and solve one-step 1st degree equations, with one variable, using the set of integers and inequalities, with one variable, using the set of whole numbers.*
>
> *Write and solve two-step 1st degree equations, with one variable, and one-step inequalities, with one variable, using the set of integers.*
>
> *Students are able to find area, volume, and surface area with whole number measurements.*

The development of area is also slow. Area problems are restricted to rectangles until seventh grade when, finally, "Students, when given the formulas, are able to find circumference, perimeter, and area of circles, parallelograms, triangles, and trapezoids (whole number measurements)." Here again, the use of whole numbers only is too restrictive. The stipulation that students be given formulas is also inappropriate. It would be a simple matter for them to memorize the relevant formulas for the geometric figures listed here. But more importantly, the standards give no indication that students should *deduce* any formulas for areas, a valuable exercise in mathematical reasoning.

The high school standards are themselves vague, but are usually clarified by accompanying examples, as in this geometry standard: "Students are able to apply properties associated with circles. Example: Find measures of angles, arcs, chords, tangents, segments and secant segments." An example that does not clarify the standards can be found elsewhere in the geometry section: "Students are able to justify properties of geometric figures. Example: Write a direct proof. Make conjectures." This is too general. Little indication is given by the standards of specific theorems and results that are to be proved. For example, there is no expectation of proof of the Pythagorean Theorem or derivation of the quadratic formula indicated in this document.

Tennessee

Reviewed: For grades K-8, Tennessee has three versions of its standards. The *Mathematics Curriculum Standards*, subtitled *Standards, Learning Expectations*, *and Draft Performance Indicators*, adopted on August 31, 2001, has standards listed as "Level 1," "Level 2," and "Level 3," without explaining the meaning of these levels. There is a separate list of K-8 standards called *Accomplishments*, also adopted on August 31, 2001. Finally, there is a summary document entitled *A Blueprint for Learning: A Teacher's Guide to the Tennessee Curriculum*. This last document combines the other two versions, but the phrasing of many standards is clarified, and standards are classified into four types: I=introduced, D=developing, A=assessed on state exams, and M=mastered and maintained. We evaluated the Tennessee K-8 standards as they appear in this *Teacher's Guide*.

2005 STATE REPORT CARD

Tennessee

Clarity: 1.83	C
Content: 1.33	D
Reason: 2.00	C
Negative Qualities: 2.00	C
Final Weighted Score: 1.70	Final Grade: D

2000 Grade: **F**

1998 Grade: **C**

Tennessee's 1998 high school standards were briefly reviewed in Fordham's *The State of State Standards 2000*. At the time of this writing, a new draft set of high school standards became available, labeled *Mathematics Curriculum Standards, DRAFT: April, 2004*. This newer set of standards was due to be ratified August 27, 2004, possibly with changes. We discuss both sets of these comparable high school standards, but the numerical ratings are based on the newer 2004 draft.

More Rigor, Please

The elementary grade standards do a good job of developing place value concepts but are woefully inadequate in developing arithmetic competence. Calculators are introduced in first grade. In second grade, students are still adding only up to 20, but second-graders are unrealistically expected to "mentally calculate add [*sic*] or subtract up to 3-digit numbers." Multiplication is not introduced until third grade. There is no mention of the standard algorithms of arithmetic throughout the standards documents.

Many standards are devoted to reading, writing, and comparing numbers, estimation, and using arithmetic properties, but the main activity of arithmetic—doing calculations—gets lost. One of the few standards addressing whole-number computation, "Multiply and divide efficiently and accurately with 1-digit numbers," is phrased in such a way as to avoid requiring that students memorize the single-digit multiplication facts. In fifth grade, whole-number computation is compressed into a single, vague, all-encompassing standard: "Add, subtract, multiply, and divide whole numbers, fractions, and decimals." This standard does not specify *how* these computations are to be done, and it is preceded by a standard that calls upon students to select appropriate methods and tools for computations. Calculators are among the suggested tools, and students decide what is appropriate. Thus, students are not actually required to be able to do these computations by hand.

This is important. By sixth grade, students should be able to add, subtract, multiply, and divide multi-digit numbers *by themselves*, without the aid of teachers or calculators. Performing computations with large numbers—done by hand, mentally when feasible, and in the course of doing multi-step problems—solidifies understanding of arithmetic and gives students confidence in arithmetic. This groundwork, much needed in preparation for algebra, is largely missing in the Tennessee standards.

The geometry standards do a good job of introducing the number line and the coordinate plane, but they overemphasize identification of figures and shapes. Geometry begins when one quantifies shapes by measuring lengths and angles and uses deduction to find other measurements. Standards of that type do not appear until eighth grade. The sixth-grade standard "Describe similarity and congruence" focuses on talking about mathematics, rather than knowing precise definitions and using them to solve geometric problems. Likewise, the standard, "Use visualization and special reasoning to solve real-world problems," which appears for all of the grades 5-8, has no specific connection to geometry.

These shortcomings are partially rectified by several good geometry standards for eighth grade. Some of them, such as "Determine the measure of an angle of a triangle given the measures of the other two angles," should have appeared in earlier grades. Eighth-graders are still plotting points in the coordinate plane (a prosaic activity begun in fourth grade), and must deal with the vague and inflated standard, "Recognize and apply geometric ideas and relationships such as tessellations in areas outside the mathematics classroom (e.g., art, science, everyday life)."

The algebra standards follow a similar pattern. After a good beginning (letters are introduced in fourth grade), the standards advance at much too slow a pace. In eighth grade, the focus is on topics that should be covered in grades 6 and 7, and even those are covered inefficiently. For example, eighth-graders are still not expected be able to solve and graph linear equations whose coefficients are fractions.

In both its 1998 and 2004 draft high school standards, Tennessee takes the peculiar approach of including five main strands (number sense, measurement, algebra, geometry, and data analysis and probability) in many of its courses, including Algebra I, Geometry, and Algebra II. The standards for the upper-level courses, such as Advanced Algebra with Trigonometry, Pre-Calculus, Statistics, and Calculus, do not follow this format.

As a consequence of this format in the lower-level high school courses, students find areas of circles in Algebra I and "apply the concept of rate of change to solve a real-world problem given a pattern of data" in Geometry. The lower-level high school courses omit fundamental topics. The Algebra I courses (both 1998 and 2004 versions) do not even mention the quadratic formula. The Geometry course involves no specific proofs or ruler-and-compass constructions. There is an abundance of manipulatives and projects through the Algebra II level. Beyond that, Advanced Algebra with Trigonometry, Pre-Calculus, and Calculus have solid standards, but it is far from clear how students can get the prerequisites needed for those courses.

Texas

Reviewed: *Texas Essential Knowledge and Skills for Mathematics* ("Chapter 111"). *TEKS Toolkit for Mathematics* includes a section entitled *Clarifying Activities for K-8* and a section entitled *Clarifying Activities for High School*. Texas provides standards for each of the grades K-8, and for high school courses, including Algebra I, Geometry, Algebra II, Pre-Calculus, and Mathematical Models with Applications.

2005 STATE REPORT CARD		
Texas		
Clarity: 2.67	B	
Content: 1.67	D	
Reason: 1.00	D	
Negative Qualities: 2.00	C	
Final Weighted Score: 1.80	Final Grade:	C
2000 Grade: **B**		
1998 Grade: **B**		

Since the 2000 Fordham review, the document *TEKS Toolkit for Mathematics* has become available. Written by the authors of *Texas Essential Knowledge and Skills for Mathematics*, the *Toolkit* explains that it is "helpful in putting the TEKS into a broader context." Our numerical evaluation of the Texas standards is lower than in the 2000 report, in part because of the generally low quality of the two sections we reviewed in the *Toolkit*. While it has some positive features, especially at the high school level, for the most part the *Toolkit* overemphasizes technology and marginal, time-wasting activities. For example, to support a sixth-grade standard calling upon students to "use ratios to describe proportional situations," the *Toolkit* activity is:

> *Students build models of cubes using marshmallows and toothpicks (clay and pipe cleaners or similar items will also work). Students create a table to record the materials needed to build a certain number of cubes so that they can determine the relationship between the needed materials and number of cubes built.*

Mostly Clear

The Texas standards are generally lucid, but there are exceptions. For example, an unfocused sixth-grade standard asks students to "use tables and symbols to represent and describe proportional and other relationships involving conversions, sequences, perimeter, area, etc." In grades 3-5, students are asked to "identify the mathematics in everyday situations," with no further elaboration provided.

Examples of inflation include using "geometric concepts and properties to solve problems in fields such as art and architecture" listed for both seventh-and eighth-graders, and for high school geometry:

> *Through the historical development of geometric systems, the student recognizes that mathematics is developed for a variety of purposes.*
>
> *The student compares and contrasts the structures and implications of Euclidean and non-Euclidean geometries.*

This latter standard has no place in a collection of geometry standards that only weakly develops synthetic Euclidean plane geometry. The geometry standards stipulate that:

> *Students use a variety of representations (concrete, pictorial, algebraic, and coordinate), tools, and technology, including, but not limited to, powerful and accessible hand-held calculators and computers with graphing capabilities to solve meaningful problems by representing figures, transforming figures, analyzing relationships, and proving things about them.*

The emphasis here on technology is misplaced. The standards themselves call for proofs in a generic way, but lack requirements to prove specific results. For example, the "student develops, extends, and uses the Pythagorean Theorem," but no proof is expected. Moreover, there is far too much emphasis on the use of concrete objects. For example, several standards begin with the phrase, "Based on explorations and using concrete models, the student formulates and tests conjectures about the properties of. . . ." This is followed by such topics as "parallel and perpendicular lines," "attributes of polygons and their component parts," and "attributes of circles and the lines that intersect them." Phrased this way, it is unclear what students are expected to know or do.

The Algebra I standards have a similar misplaced reliance on technology and concrete models. For example, one standard is, "The student solves quadratic equations using concrete models, tables, graphs, and algebraic methods." The overuse of tables and concrete objects to solve quadratic equations detracts from the vastly more important algebraic methods. The treatment of quadratic polynomials is weak. Students are steered away from sound mathematical reasoning by a requirement to use "patterns to generate the laws of exponents." Laws of exponents should be developed, not by appealing to patterns, but rather by using clear definitions and fundamental properties of algebra, such as the associative property of multiplication.

The Algebra II standards are better than those for Algebra I. Major topics are addressed, including complex numbers, completing the square, conic sections, and logarithms and exponentials. However, this last topic is badly presented:

> *The student develops the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses.*

This is an example of false doctrine. Students should not be asked to discover or develop definitions of standard terms. Students are entitled to clear, unambiguous definitions.

Elementary Problems

The elementary grade standards appropriately require memorization of the basic number facts, but also encourage open-ended use of technology:

> *Throughout mathematics in Kindergarten-Grade 2, students use . . . technology and other mathematical tools . . . to develop conceptual understanding and solve problems as they do mathematics.*

Similar statements are made for subsequent grade levels. A standard appearing for each of the grades K-5 is "use tools such as real objects, manipulatives, and technology to solve problems." Almost no guidance is provided for student use of technology. The only exceptions are these two fifth-grade standards:

> *Use multiplication to solve problems involving whole numbers (no more than three digits times two digits without technology).*

> *Use division to solve problems involving whole numbers (no more than two-digit divisors and three-digit dividends without technology).*

Unfortunately, these standards place no formal restriction on the use of technology; rather, they restrict paper-and-pencil calculations to whole numbers with *no more than* two and three digits.

Fractions are slowly and poorly developed. In fifth grade, students "use lists, tables, charts, and diagrams to find patterns and make generalizations such as a procedure for determining equivalent fractions" and "compare two fractional quantities in problem-solving situations using a variety of methods, including common denominators." They also "use models" to "relate decimals to fractions" and fifth-graders "model and record addition and subtraction of fractions with like denominators in problem-solving situations." There is no mention of addition and subtraction of fractions with different denominators, or of multiplication or division of fractions in the K-5 standards.

At the middle school level, the arithmetic of rational numbers is largely held hostage to manipulatives and calculators, poor preparation for high school mathematics. The standards for grades 6-8 are prefaced by this strong endorsement:

> *Throughout mathematics in Grades 6-8, students use these processes together with technology (at least four-function calculators for whole numbers, decimals, and fractions) and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.*

Sixth-graders are still using manipulatives instead of symbolic notation when they "model addition and subtraction situations involving fractions with objects, pictures, words, and numbers." Seventh-graders "convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator," yet the TEKS does not make clear what students are expected to be able to do without calculators.

Conflating geometry with statistics, sixth-graders are expected to "generate formulas to represent relationships involving perimeter, area, volume of a rectangular prism, etc., from a table of data." Unarticulated is any expectation for students to understand a logical progression of formulas for areas of basic polygons by relating areas of triangles to areas of rectangles, parallelograms, and trapezoids in a coherent way.

Utah

Reviewed: Utah's *Core Standards*, revised May 2003. Utah provides standards for each of the grades K-6 along with standards for courses for grades 7-12. The standards documents describe course sequences leading up to Advanced Placement Calculus and Advanced Placement Statistics. There is an optional intervention course, Math 7, and then a conventional sequence that proceeds through Pre-Algebra, Elementary Algebra, Geometry, Intermediate Algebra, and Pre-Calculus. In another sequence, Pre-Algebra is followed by Applied Mathematics I and II.

Utah	**2005 STATE REPORT CARD**
Clarity: 1.83	C
Content: 1.17	D
Reason: 0.50	F
Negative Qualities: 1.00	D
Weighted Score: 1.13	Final Grade: D
2000 Grade: **B**	
1998 Grade: **B**	

Mathematical content in Utah's standards is undermined by an insistence on the use of manipulatives to carry out calculations, at all levels, except in the advanced high school courses. The section "Key Principles and Processes for Teaching Mathematics for Deep Understanding" that precedes middle and high school course standards includes this statement:

> *Students need to know and be able to use basic mathematical facts and procedures. However, current research makes clear that* how *mathematics is taught is as important or more important than the mathematical concepts being taught.* [emphasis in original]

Utah integrates dubious pedagogical directives into its content standards. Consider the Elementary Algebra standards listed under Objective 2.2, "Evaluate, solve, and analyze mathematical situations using algebraic properties and symbols":

> *Solve multi-step equations and inequalities:*
>
> *a. Numerically; e.g., from a table or guess and check.*
>
> *b. Algebraically, including the use of manipulatives.*
>
> *c. Graphically.*
>
> *d. Using technology.*
>
> *Solve systems of two linear equations or inequalities:*
>
> *a. Numerically; e.g., from a table or guess and check.*
>
> *b. Algebraically.*
>
> *c. Graphically.*
>
> *d. Using technology.*

The ability to solve algebraic equations and inequalities is overwhelmingly important for algebra students, and it should not be done with manipulatives, graphs, calculators, or by guessing, but by the systematic use of properties of equality, inequality, and the field properties of the rational and real number systems. Under Elementary Algebra Objective 3.3, "Solve problems using visualization, spatial reasoning, and geometric modeling," these standards are listed:

> *3. Illustrate multiplication of polynomials using area models, e.g.,*
>
> $(a + b)^2$, $x(x + 2)$, *or* $(x + a)(x + b)$.
>
> *4. Factor polynomials using area models:*
>
> *a. To identify the greatest common monomial factor.*
>
> *b. Of the form* ax2 + bx + c *when a = 1.*

The principal tool for multiplying and factoring polynomials is the distributive property, not "area models." But the distributive property is not mentioned in the Elementary Algebra standards. We see in these examples the emphasis of pedagogy over content found throughout the Utah standards.

The high school Geometry standards focus on trigonometry and analytic geometry, valuable topics, but there is no mention of proofs for specific theorems in Euclidean geometry, a major failing. The Intermediate Algebra standards include, for the first time, requirements for students to complete the square of quadratic polynomials, an important skill. Some attention is given to matrix algebra, but the far more important algebra of rational functions is missing. However, the Pre-Calculus standards are mostly solid.

Good Start, Bad Close

The early grade standards offer a systematic build-up of place value and counting, but the elementary grade standards are inadequate. Already in Kindergarten there is confusion between mathematics and the study of patterns. According to Standard II, "Students will identify and use patterns to represent mathematical situations." This is followed by, "Use patterns to count orally from 1 to 20 and backward from 10 to 0." How can patterns be used to count backward from ten to zero? No explanation is given.

According to Standard V, kindergartners will "understand basic concepts of probability," and this is followed by a directive for kindergartners to "Relate past events to future events (e.g., The sun set about 6:00 last night, so it will set about the same time tonight)." How this might contribute to an understanding of probability by kindergartners is not explained. More importantly, without facility with fractions, students cannot learn probability. Probability standards do not belong in the lower grades, and certainly not in Kindergarten.

The elementary grade standards do not require students to memorize the basic number facts. Instead, second-graders "compute accurately with basic number combinations for addition and subtraction facts to eighteen." While it is desirable for students to compute accurately, it is also important that they memorize such facts as *9 + 8 = 17*, without the necessity to compute it each time it is needed. Two third-grade standards for single-digit multiplication are:

> *Find the products for multiplication facts through ten times ten and describe the process used.*

> *Model multiplication of a one-digit factor by a one-digit factor using various methods (e.g., repeated addition, rectangular arrays, manipulatives, pictures) and connect the representation to an algorithm.*

Both standards work implicitly against memorization of the multiplication facts. In the first of the above standards, students are required to "describe the process used" to find, for example, *6 x 8*, while in the second, they must find an unidentified algorithm to produce the answer. Students should memorize the fact that *6 x 8 = 48*, rather than having to search for elusive algorithms that will yield that result.

Elementary grade standards repeatedly call upon students to "describe the process used" to find sums, differences, products, and quotients of whole numbers and decimals. No reference is made to the standard algorithms of arithmetic, a serious deficiency. Calculators are introduced in second grade, and in each of the grades 2 through 6, we find the standard,

> *Use a variety of methods and tools to facilitate computation (e.g., estimation, mental math strategies, paper and pencil, calculator).*

Fractions and decimals receive considerable attention, but mainly through the use of manipulatives. For example, fourth-graders "find equivalent fractions for one-half, one-third, and one-fourth using manipulatives and pictorial representations," and fifth-graders

> *Represent commonly used fractions as decimals and percents in various ways (e.g., objects, pictures, calculators).*

The sixth-grade standards do call for students to compute with positive rational numbers, but the role of manipulatives and calculators is significant and the standards do not identify what students should be able to do with pencil and paper. Overall, the Utah standards fail to develop arithmetic adequately.

Some standards in the elementary grades for algebra are sound, and geometry is introduced early on, but the focus is on naming things: parallelograms in second grade, rhombuses, trapezoids, and kites in fourth grade. Probability is introduced without its major prerequisite, the arithmetic of fractions.

Vermont

Reviewed: *Framework of Standards and Learning Opportunities*, Fall 2000, provides standards for the following bands of grades: Pre-K to 4; 5-8; and 9-12. A newer document, *Grade Expectations for Vermont's Framework of Standards and Learning Opportunities*, Spring 2004, provides grade-level expectations (GLEs) for each of the grades K-8 and one unspecified grade at the high school level. These GLEs were developed jointly by Vermont, New Hampshire, and Rhode Island, and they define the content of the New England Common Assessment Program (NECAP) for grades 3-8 in these states. According to *Grade Expectations for Vermont's Framework of Standards and Learning Opportunities*, the GLEs are more specific statements for the Vermont standards that meet the No Child Left Behind Act's requirements for test development.

2005 STATE REPORT CARD

Vermont

Clarity: 1.33	D
Content: 1.00	D
Reason: 0.67	F
Negative Qualities: 2.00	C
Weighted Score: 1.20	Final Grade: D

2000 Grade: C

1998 Grade: C

Vermont's standards are the best of the three states that jointly developed grade-level expectations associated with the New England Common Assessment Program, but that is not saying much. In spite of the assertion that the "GLEs are more specific statements for the Vermont standards," these documents are not consistent with each other. According to the *Framework*, students in grades Pre-K to 4:

> *7.6.b. begin to use simple concepts of negative numbers, properties of numbers (e.g., prime, square, composite, associative, commutative, distributive), three-digit and larger multipliers and divisors, rates, fractions, decimals, and percents.*

However, this standard is not supported by the GLEs. For example, the word "integer" is first mentioned in this poorly worded sixth-grade standard:

> *Demonstrates understanding of the relative magnitude of numbers by ordering or comparing numbers with whole-number bases and whole-number exponents (e.g., 3^3, 4^3), integers, or rational numbers within and across number formats (fractions, decimals, or whole-number percents from 1 to 100) using number lines or equality and inequality symbols.*

The following fourth-grade GLE also falls short of Framework Standard 7.6.b.:

> *Accurately solves problems involving multiple operations on whole numbers or the use of the properties of factors and multiples; and addition or subtraction of decimals and positive proper fractions with like denominators. (Multiplication limited to 2 digits by 2 digits, and division limited to 1 digit divisors.)*

In the elementary grade standards, students are not required to use or understand the conventional arithmetic algorithms or to memorize the basic number facts. Instead, the fourth-grade student,

> *Mentally adds and subtracts whole numbers through twenty and multiplies whole numbers through twelve with accuracy.*

It is reasonable to ask students to be able to mentally add any two whole numbers up to 20, and to mentally multiply two whole numbers up to 12, but there is also great value in memorizing the fact that *9 x 7 = 63*, for example, without the need to calculate that result each time it is needed. Further compounding the shortcomings of the GLEs, the Framework asks Pre-K to fourth-grade students to "add, subtract, multiply, and divide whole numbers, with and without calculators." Neither the *Framework* nor the GLEs make clear when calculator use is appropriate for early grade students.

All Mixed Up

The GLEs frequently suffer from convoluted writing, as illustrated by this fifth-grade standard:

> *Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from 0 to 9,999,999 through equivalency, composition, decomposition, or place value using models, explanations, or other representations; positive fractional numbers (proper, mixed number, and improper) (halves, fourths, eighths, thirds, sixths, twelfths, fifths, or powers of ten [10, 100, 1000]), decimals (to thousandths), or benchmark percents (10%, 25%, 50%, 75%, or 100%) as a part to whole relationship in area, set, or linear models using models, explanations, or other representations.**

The asterisk in the last line references a footnote that places near-incomprehensible restrictions on the rational numbers that students consider:

> **Specifications for area, set, and linear models for grades 5-8: Fractions: The number of parts in the whole are equal to the denominator, a multiple of the denominator, or a factor of the denominator. Percents: The number of parts in the whole is equal to 100, a multiple of 100, or a factor of 100 (for grade 5); the number of parts in the whole is a multiple or a factor of the numeric value representing the whole (for grades 6-8). Decimals (including powers of ten): The number of parts in the whole is equal to the denominator of the fractional equivalent of the decimal, a multiple of the denominator of the fractional equivalent of the decimal, or a factor of the denominator of the fractional equivalent of the decimal.*

The *Framework* calls upon students in grades 5-8 to "multiply and divide rational (fractional) numbers," but these fundamental skills are not explicitly mentioned in the GLEs. One finds instead the ambiguously worded sixth-grade standard:

> *Accurately solves problems involving single or multiple operations on fractions (proper, improper, and mixed), or decimals; and addition or subtraction of integers; percent of a whole; or problems involving greatest common factor or least common multiple.*

How does a sixth-grade teacher interpret this directive? Do students in this grade learn to multiply and divide fractions, or is that left for later grades?

The K-8 GLEs overemphasize probability and statistics. High school students are expected to work with normal distributions, and eighth-graders estimate lines of best fit. To develop these topics properly is college-level mathematics, and to do it other ways is not mathematics.

Starting in first grade, students are expected to analyze sample spaces in which outcomes may or may not be equally likely:

> *For a probability event in which the sample space may or may not contain equally likely outcomes, use experimental probability to describe the likelihood or chance of an event (using "more likely," "less likely").*

The probability of an event is a number between zero and one. It makes no sense to discuss probability until students have at least a working knowledge of fractions. The following eighth-grade standard continues the chain of standards that begins with the previous one, and is an example of false doctrine:

> *For a probability event in which the sample space may or may not contain equally likely outcomes, determine the possible outcomes by either sample space (organized list, table, tree model, area model) or Fundamental Counting Principle and determine the theoretical probability of that event as a ratio of*

favorable outcomes to possible outcomes. Express the ratio as a fraction, decimal, or percent.

In general, the probability of an event is *not* a "ratio of favorable outcomes to possible outcomes" when outcomes in the sample space are not equally likely.

These high school expectations are evidently intended as minimal expectations, but it is a weakness of the Vermont standards that no discussion of high school mathematics beyond that minimum is given.

Virginia

Reviewed: *Mathematics Standards of Learning for Virginia Public Schools*, approved in October 2001. The document provides standards for each of the grades K-8, and course standards for Algebra I, Geometry, Algebra II, Trigonometry, Computer Mathematics, Discrete Mathematics, Mathematical Analysis, Advanced Placement Calculus, and a course combining the content of Algebra II and Trigonometry. *The Mathematics Standards of Learning Curriculum Framework*, published in 2002, includes elaboration of the standards and teacher notes.

2005 STATE REPORT CARD

Virginia

Clarity: 2.83	B
Content: 2.00	C
Reason: 1.50	D
Negative Qualities: 1.50	D
Weighted Score: 1.97	Final Grade: C
2000 Grade: **B**	
1998 Grade: **B**	

Virginia's 2001 and 2002 revision to its venerable *Standards of Learning* in math have caused the state's grade to slip somewhat.

The elementary grade standards have some commendable features. The standards on measurement, which cover both common and metric units, are well organized. Virginia students, unlike students in many other states, are expected to memorize the basic number facts. But the elementary standards fall short in the way they treat calculators, word problems, and algorithms. There is also a lack of coordination in the development of fractions and decimals.

Too Much Technology

However, the Virginia standards rely excessively on calculators. Beginning in Kindergarten,

> *The student will investigate and recognize patterns from counting by fives and tens to 30, using concrete objects and a calculator.*

Requiring students to use calculators even before they have a firm grasp of how to count and what addition means is counterproductive. Directives to use calculators throughout the elementary school grades in the Virginia standards undermine what would otherwise be credible arithmetic standards. An example is this fourth-grade standard:

> *The student will compare the value of two decimals, using symbols (<, >, or =), concrete materials, drawings, and calculators.*

Part of the genius of the base-ten number system is that decimals, i.e., mixed numbers expressed in decimal notation, can be compared at a glance. The use of calculators to decide which of two given decimals is the larger has no mathematical or educational justification. Several standards, such as this one for fourth grade, direct *students* to decide when it is appropriate to use calculators:

> *The student will add and subtract whole numbers written in vertical and horizontal form, choosing appropriately between paper and pencil methods and calculators.*

Guidance on appropriate calculator use is largely missing from the *Framework* and standards. A particularly egregious example is Standard 2.5, which begins by stating that second-graders should be able to "count forward by twos, fives, and tens to 100, starting at various multiples of 2, 5, or 10." But the rest of the sentence then undermines this sound directive, with the phrase, "using mental mathematics, paper and pencil, hundred chart, calculators and/or concrete objects, as appropriate." Is it "and" or "or"? It is not *ever* appropriate for second-graders to use calculators to count by twos, fives, or tens?

The inappropriate use of calculators continues at the middle school level. One standard states that "calculators will be used to develop exponential patterns." The goal in grade 6 should be to understand laws of exponents. Calculators are of no help for this purpose; after all, exponents are used to *avoid* unnecessary computations. Laws of exponents can be far better illustrated without calculators, using clear definitions, familiar properties of arithmetic, and simple hand calculations with small numbers.

Word problems, especially multi-step word problems, are essential to develop conceptual understanding of arithmetic. Despite boilerplate language stating that "problem solving has been integrated throughout," the Virginia standards give scant attention to word problems. However, two sixth-grade standards do call upon students to solve multi-step problems.

Learning the conventional arithmetic algorithms is essential for understanding arithmetic. The Virginia standards call for numerical calculations without requiring students to use or understand the standard algorithms, as in this fifth-grade standard:

> *The student will find the sum, difference, and product of two numbers expressed as decimals through thousandths, using an appropriate method of calculation, including paper and pencil, estimation, mental computation, and calculators.*

A teacher note in the *Framework* elaborates:

> *There are a variety of algorithms for division such as repeated multiplication and subtraction. Experience with these algorithms may enhance understanding of the traditional long-division algorithm.*

This statement refers to long division without indicating that students are actually expected to *learn* long division.

Distorted Development

The standard cited above helps to illustrate the lack of coordination between Virginia's development of decimal arithmetic and fraction arithmetic. The following fifth-grade standard places a restriction on denominators of fractions:

> *The student will add and subtract with fractions and mixed numbers, with and without regrouping, and express answers in simplest form. Problems will include like and unlike denominators limited to 12 or less.*

Comparing these two fifth-grade standards, teachers might wonder how to explain to their students the meaning of the equation $0.01 + 0.02 = 0.03$ without making reference to fractions whose denominators exceed 12. This unnecessary restriction on the size of denominators of fractions (12 or less) continues through the seventh grade.

Progress in algebra is slow in middle school. By the end of the eighth grade, students solve simple linear equations or graph them using a table. But students are not introduced to the concept of slope, nor do they necessarily see the different ways of writing the equation of a line.

References to manipulatives are sprinkled into the standards in ways that confuse teaching devices with the skills to be learned (e.g.: "model and solve algebraic equations using concrete materials"). The use of manipulatives persists long after students should have progressed to higher conceptual levels. For example, in sixth grade, at the point where students should be solidifying their knowledge of arithmetic, we find the standard "compare and order whole numbers, fractions, and decimals using concrete materials, drawings or pictures, and mathematical symbols."

The standards for Trigonometry, Mathematical Analysis, and Calculus are well written and appropriate for college-bound students. However, the Algebra I, Geometry, and Algebra II standards have several deficiencies. The enormous emphasis on graphing calculators and manipulatives undermines the development of algebra skills and understanding. For example, understanding the effect of changes in the slope on the graph of a line is best done by hand rather than by graphing calculator, as called for in Standard A.7. Factoring simple binomials and trinomials should be done by hand, not with a calculator, as called for by Standard A.12. Students should not be required to use manipulatives in an Algebra I course, as they are in Standard A.3 for the purpose of solving equations, and Standard A.11 to carry out polynomial arithmetic. Algebra is an extension of arithmetic that is learned by building on knowledge of arithmetic, not by returning to the pre-arithmetic level.

Virginia's Geometry standards are also deficient. High school geometry provides an important opportunity for students to learn deductive reasoning—formulating and justifying assertions of the form, "If statements A and B are true, then statement C must *necessarily* be true." Virginia's high school geometry course begins with an excellent standard that prepares the ground for deductive reasoning. But deductive reasoning is nearly lost in the subsequent standards, which include time-consuming diversions lacking depth and specificity (e.g., "Tessellations and tiling problems will be used to make connections to art, construction, and nature," or "Models and representations will include scale drawings, perspective drawings, blueprints, or computer simulations"). There is almost no attention given to the logical structure of geometry. Some of the most important facts obtained by deductive reasoning are missing, including the fact that the sum of the interior angles of a triangle is 180 degrees, the fact that angles inscribed in semi-circles are right angles, and the proof of the Pythagorean Theorem.

Washington

Reviewed: *Mathematics, K-10 Grade Level Expectations: A New Level of Specificity*, 2004. Washington provides standards for each of the grades K-8 and one set of standards for grades 9 and 10.

2005 STATE REPORT CARD

Washington

Clarity: 0.33	**F**
Content: 1.00	**D**
Reason: 0.50	**F**
Negative Qualities: 0.00	**F**
Weighted Score: 0.57	Final Grade: **F**

2000 Grade: **F**

1998 Grade: **F**

Standards for each grade are arranged in five strands: 1. The student understands and applies the concepts and procedures of mathematics. 2. The student uses mathematics to define and solve problems. 3. The student uses mathematical reasoning. 4. The student communicates knowledge and understanding in both everyday and mathematical language. 5. The student understands how mathematical ideas connect within mathematics, to other subject areas, and to real-life situations.

A good first step toward improving Washington's failed standards would be to eliminate all but the first of the five strands identified in the grade-level expectations. The other four express laudable but inherently vague goals, and the specific standards listed under them are often of such low quality that they are likely to create more confusion and frustration than enlightenment. Though uneven in quality, the first strand includes credible benchmarks, particularly in the middle school grades.

Overall, the Washington standards are poorly written and needlessly voluminous. Some standards, such as these for grades 5 and 9/10 respectively, are difficult even to understand:

> *Translate a situation involving two alternating arithmetic operations into algebraic form using equations, tables, and graphs (e.g., a snail crawls up 3 feet each day and slides back 2 feet each night).*
>
> *Determine when two linear options yield the same outcome (e.g., given two different investment or profit*

options, determine when both options will yield the same result).

Other standards have little to do with mathematics, such as the following from different grade levels:

> *Explain or show how height and weight are different.*
>
> *Explain or show how clocks measure the passage of time.*
>
> *Explain how money is used to describe the value of purchased items.*
>
> *Determine the target heart zone for participation in aerobic activities.*
>
> *Determine adjustments needed to achieve a healthy level of fitness.*
>
> *Explain why formulas are used to find area and/or perimeter.*
>
> *Explain a series of transformations in art, architecture, or nature.*

The standards also include classroom activities that are untestable and only marginally related to mathematics, such as:

> *Recognize the contributions of a variety of people to the development of mathematics (e.g., research the concept of the golden ratio).*

Problematic Problem-Solving

The standards devoted to problem-solving are of especially low quality. Instead of specifying types of problems that students should be able to solve, the Washington standards give long, repetitive lists of vague, generic tasks (e.g., "Gather and organize the necessary information or data from the problem," "Use strategies to solve problems," "Describe and compare strategies and tools used," "Generate questions that could be answered using informational text"). These sections are both misleading and useless: misleading because one does not learn how to solve mathematics problems by following the outlines presented, and useless because they give no hint about which types of problems students are expected to solve.

In fact, these sections focus on *talking about solving problems*, rather than actually solving them. Only a single example problem is provided for each grade, each of which displays a fundamental misunderstanding of the nature of mathematical problems. A mathematics problem should be clearly stated; it should contain all of the information needed to solve it; and it should have a definite answer. The sample problems in the Washington standards violate all three of these criteria. The following first-grade problem is typical:

> *A classroom is presenting a play and everyone has invited two guests. Enough chairs are needed to seat all the guests. There are some chairs in the classroom.*

This is not a mathematics problem at all, nor can it be turned into one until more information is provided. The sample problem for grades 9 and 10 is arguably even worse. That problem asks if it is "reasonable to believe that the women will run as fast as the men" in the Olympics. A list of running times of men and women, for an unspecified distance, is then provided for several years of Olympic games. No further information is provided. The framework implies that problems like these are good starting points for classroom discussions about solving problems. But such problems risk miseducating students to believe that mathematics itself is ambiguous, a matter of opinion, and without definite answers.

Too Much Technology

In the elementary grades, students are expected to memorize the basic number facts, a positive feature. However, student-invented algorithms and calculators are strongly emphasized throughout. The framework includes explicit requirements for calculator use beginning in first grade. Fourth-graders are expected to

> *use calculators to compute with large numbers (e.g., multiplying two* [sic] *digits times three digits; dividing three or four digits by two digits without remainders).*

Fourth-graders should be able to multiply two-digit numbers by three-digit numbers, and to divide numbers by hand, using the standard algorithms. Students who do not master those skills have a difficult time with middle school mathematics.

Fifth grade students are expected to "demonstrate the effect of multiplying a whole number by a decimal number" before they are given a general definition of fraction multiplication, a topic that appears for the first time in the sixth-grade standards. Further compounding this deficiency is this fifth-grade standard:

> *Use calculators to multiply or divide with two decimal numbers in the hundredths and/or thousandths place.*

Fifth-graders are thus required to use calculators to multiply decimal numbers before they are even exposed to the meaning of fraction multiplication. What does it mean to multiply two fractions or, in particular, two decimals? The answer comes a year later. This is rote use of technology without mathematical reasoning. A fundamental misunderstanding is promoted by this fifth-grade standard:

> *Explain how the value of a fraction changes in relationship to the size of the whole (e.g., half a pizza vs. half a cookie).*

This confuses fractions, which are numbers, with quantities, which are numbers with units (such as "3 lbs."). If we change the quantity "half a pizza" to "half a cookie" we are changing the unit, not the fraction. This is not a quibble; it is a fundamental misinterpretation of the meaning of fractions.

Throughout the grade levels there is too much emphasis on patterns, probability, and data analysis to the exclusion of more important topics. The grade 9/10 standards are weak. The algebra standards involve little more than linear equations: Quadratic equations are not even mentioned and the concept of function receives almost no attention. Little is done with proofs or geometric reasoning.

West Virginia

Reviewed: *Mathematics Content Standards K-12*, July 1, 2003. West Virginia provides standards for each of the grades K-8 arranged in five strands (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability), and standards for each of 11 high school courses.

2005 STATE REPORT CARD

West Virginia

Clarity: 2.00	**C**
Content: 2.50	**B**
Reason: 3.00	**B**
Negative Qualities: 1.75	**C**
Weighted Score: 2.35	Final Grade: C

2000 Grade: **B**

1998 Grade: **B**

West Virginia's standards have fallen in quality with this unwieldy revision. The document begins by listing 17 overarching standards that are intended to apply to all grades. Those 17 standards are repeated in each grade, followed by the actual standards for that grade. The general standards may have provided thematic guidance for the authors, but they serve no purpose in the grade-level standards, and it is confusing to see them listed above the actual standards. West Virginia also defines five levels of performance: Distinguished, Above Mastery, Mastery, Partial Mastery, and Novice. The framework attempts to define these levels by including "performance descriptors" for each strand in each grade K-8. However, these descriptors are lengthy, repetitive, and unwieldy.

The elementary standards require students to memorize the basic number facts and to perform whole number, fraction, and decimal calculations. For example, the fourth-grade standards ask them to multiply and divide three-digit numbers by one- and two-digit numbers both as isolated problems and in the course of story problems. These are appropriate standards, but their effectiveness is undermined by the fact that none of the West Virginia standards calls upon students to use or understand the standard algorithms of arithmetic.

Further weakening the elementary grade arithmetic standards is the blanket statement, "West Virginia teachers are responsible for analyzing the benefits of technology for learning and for integrating technology appropriately in the students' learning environment," which appears in the introduction to the standards in every grade. This statement instructs each teacher independently to decide whether calculators are to be used to meet standards. In general, it is a good idea to give teachers latitude in deciding how to meet standards, but in this case such latitude has potentially negative consequences. It is easy to teach students to "multiply and divide 3-digit numbers by 1 and 2-digit numbers" on a calculator—because doing so requires no understanding of place value or multiplication. Unfortunately, such defective instruction would be consistent with West Virginia's elementary grade standards.

Inconsistent Standards

The elementary grade geometry standards and measurement standards are appropriate, generally well written, and thorough. However, some of them are too vague, such as the fourth-grade measurement standard, "understand appropriate grade level conversions within a system of measure." The algebra standards display the weaknesses endemic in standards that include an algebra strand extending all the way down to Kindergarten, notably a tedious emphasis on patterns, as in these third-grade standards:

- *Analyze and complete a geometric pattern.*
- *Identify and write number patterns of 3's and 4's.*
- *Identify and write the rule of a given pattern.*

"Geometric patterns" are not defined or explained; it is unclear what is meant by "patterns of 3's and 4's"; and the beginning of a pattern never has a unique rule. However, the elementary grade algebra standards introduce the use of letters for unknown numbers in preparation for the later study of algebra, a positive feature.

The middle school standards cover middle school topics such as ratios, volumes, and linear equations well, and build a good foundation for high school algebra and geometry.

The standards for each grade and course begin with an introductory paragraph. As noted above, these paragraphs allow teachers to decide the extent of technology use in their courses, a serious flaw. Aside from that, these paragraphs are straightforward summaries of the mathematics addressed in grades K-5. However, starting in sixth grade, the introductory paragraphs increasingly become statements of educational doctrine and prescriptions for teaching methods. For example, the introductory paragraph for eighth grade instructs the reader that, "Lessons involving cooperative learning, manipulatives, or technology will strengthen students' understanding of concepts while fostering communication and reasoning skills." It is likely that eighth-grade students will learn more by building on their previous knowledge of mathematics, not starting from scratch with manipulatives. Mathematics owes its power and breadth of utility to abstraction. The overuse of manipulatives in the higher grades works against sound mathematical content and instruction.

High School Standards Mostly Solid

Of West Virginia's 11 high school courses, some are clearly designed for college-bound students, while others are remedial. The framework thus provides flexibility for schools to offer a variety of courses based on student needs. Aside from the introductory paragraphs, the standards for Algebra I, Algebra II, Geometry, Trigonometry, Probability and Statistics, and Pre-Calculus are generally sound, well written, and appropriate. However, there are shortcomings. Standards such as these, that require students to "explore" or "investigate," cannot be meaningfully assessed:

> *Explore the relationship between angles formed by two lines cut by a transversal when lines are and are not parallel, and use the results to develop methods to show parallelism.*
>
> *Investigate measures of angles formed by chords, tangents, and secants of a circle and the relationship to its arcs.*

Probability and statistics standards are overemphasized in the high school standards (except, of course, in the standards for the Probability and Statistics course, where they belong). These probability and statistics standards are out of place among the Algebra I standards:

> *Perform a linear regression and use the results to predict specific values of a variable, and identify the equation for the line of regression.*

> *Use process (flow) charts and histograms, scatter diagrams and normal distribution curves.*

Throughout the document one finds poorly worded standards. For example, "Represent the idea of a variable as an unknown quantity using a letter" would be better expressed as, "Use letters to represent unknown numbers." The fifth-grade standard, "Model multiplication and division of fractions to solve the algorithm," would be improved by wording such as, "Use area pictures to model multiplication and division of fractions. Multiply fractions using the definition. Divide fractions using the 'invert-and-multiply' algorithm."

Another example of poor wording is the seventh-grade standard, "use the concept of volume for prisms, pyramids, and cylinders as the relationship between the area of the base and the height." These are minor problems, but the West Virginia standards should have been proofread, at least, by someone with a solid knowledge of mathematics.

Wisconsin

Reviewed: *Wisconsin Model Academic Standards for Mathematics*, January 13, 1998. Wisconsin provides standards for the band of grades K-4, 5-8, and 9-12. This document also includes a glossary.

2005 STATE REPORT CARD

Wisconsin

Clarity: 1.67	D
Content: 1.67	D
Reason: 1.00	D
Negative Qualities: 1.50	D
Weighted Score: 1.50	Final Grade: D
2000 Grade: C	
1998 Grade: C	

Wisconsin's grade has dropped, despite its not having new standards, because of our heightened emphasis on content. At the outset, it should be said that Wisconsin's standards have an unusual and commendable feature: the directive to "read and understand mathematical texts." Students need to learn arithmetic, algebra, geometry, and other parts of mathematics, but they also benefit from learning to read and comprehend math books. Doing so requires the use of mathematical reasoning.

Overall, however, mathematical reasoning is only weakly supported in this short standards document. The "Mathematical Process" standards urge students to "use reasoning abilities" to do such things as "perceive patterns," "identify relationships," "formulate questions for further exploration," etc. Yet these standards are completely separate from the content standards. A particular "Mathematical Process" standard for eighth grade deserves comment:

> *Analyze non-routine problems by modeling, illustrating, guessing, simplifying, generalizing, shifting to another point of view, etc.*

A nearly identical standard appears for the end of twelfth grade. Certainly the abilities called for here are desirable, but there is no analogous requirement to analyze, let alone solve, the far more important routine problems that build skills and consolidate understanding of mathematical concepts. Novelty for its own sake is of little value.

The Wisconsin elementary grade standards require the memorization of basic number facts, but there is no requirement for students to learn the standard algorithms of arithmetic, and calculators are to be used for whole-number calculations.

Guidance, Please

The terse middle school standards require computations with rational numbers, but students evidently create their own algorithms rather than learn the powerful standard algorithms, as indicated in this standard:

> *In problem-solving situations, select and use appropriate computational procedures with rational numbers such as*
>
> - *calculating mentally*
> - *estimating*
> - *creating, using, and explaining algorithms*
> - *using technology (e.g., scientific calculators, spreadsheets)*

The middle school algebra standards are broad but vague. For example:

> *Work with algebraic expressions in a variety of ways, including*
>
> - *using appropriate symbolism, including exponents and variables*
> - *evaluating expressions through numerical substitution*
> - *generating equivalent expressions*
> - *adding and subtracting expressions*

There is no guidance from this standard, or others, about the types of algebraic expressions with which students should work. Are they expected to work with polynomials, rational expressions, expressions with radicals, or only linear functions? Teachers must decide, and they can make a variety of different decisions consistent with these standards.

Many standard topics are missing from the high school standards, including any reference to the binomial theorem, the arithmetic of rational functions, completing the square of quadratic polynomials, and conic sections. Trigonometry and the Pythagorean Theorem receive little attention.

Wyoming

Reviewed: *Wyoming Mathematics Content and Performance Standards*, Adopted July 7, 2003. Wyoming provides standards for each of the grades K-8 and grade 11. Each standard is classified by content strand: Number Operations and Concepts; Geometry; Measurement; Algebraic Concepts and Relationships; and Data Analysis and Probability. Some of the strands at particular grade levels include "Action Snapshots," which give classroom activities aligned with standards, or which elaborate on the meanings of standards. For each grade and strand, there is a description of four levels of student performance: Advanced, Proficient, Basic, and Below Basic.

2005 STATE REPORT CARD

Wyoming

Clarity: 1.00	**D**
Content: 0.83	**F**
Reason: 0.00	**F**
Negative Qualities: 2.25	**C**
Weighted Score: 0.98	Final Grade: **F**
2000 Grade: **D**	
1998 Grade: **-**	

Wyoming slips into failing territory with these vague standards, which are difficult to recognize as a useful guide to instruction or assessment. Each strand for each grade, starting in Kindergarten, carries the same directive: "Students communicate the reasoning used in solving these problems. They may use tools/technology to support learning." Teachers are evidently free to incorporate calculators and other forms of technology as they see fit. Redundancy from one grade level to the next is illustrated by the following geometry standards:

Kindergarten

Students select, use, and communicate organizational methods in a problem-solving situation using geometric shapes.

Grade 1

Students select, use, and communicate organizational methods in a problem-solving situation using 2- and 3-dimensional geometric objects.

Grade 2

Students select, use, and communicate organizational methods in problem-solving situations with 2- and 3-dimensional objects.

Grade 3

Students select, use, and communicate organizational methods in problem-solving situations appropriate to grade level.

No elaboration of these directives is provided.

Hazy Expectations

Students in the elementary grades are not required to memorize the basic number facts. Instead, fourth-graders "demonstrate computational fluency with basic facts (add to 20, subtract from 20, multiply by 0-10)." Computational fluency is defined in the Action Snapshot as follows:

Computational fluency is a connection between conceptual understanding and computation proficiency. Conceptual understanding of computation is grounded in mathematical foundations such as place value, operational properties, and number relationships. Computation proficiency is characterized by accurate, efficient, and flexible use of computation for multiple purposes.

Similar language appears in the standards for other grade levels. There is no mention of the standard algorithms of arithmetic for whole-number or decimal calculations. Fourth-grade students choose their own procedures and "explain their choice of problem-solving strategies and justify their results when performing whole number operations in problem-solving situations." A fourth-grade Action Snapshot elaborates:

One student might add four sets of 6 apples to get 24, and another student might multiply 6 times the 4 sets to get the same results. The explanations should represent their procedure and results. Children should know multiple strategies, but do not have to demonstrate them all in one problem (for example, front end loading addition).

The three standards below constitute the entire algebra strand for fourth grade:

1. *Students recognize, describe, extend, create, and generalize patterns by using manipulatives, numbers, and graphic representations.*
2. *Students apply knowledge of appropriate grade level patterns when solving problems.*
3. *Students explain a rule given a pattern or sequence.*

Probability standards are given before standards mentioning fractions even appear. A first-grade Action Snapshot recommends that students "use spinners, coins, or dice." But the first mention of the word "fraction" is in the fourth-grade standards.

Low Expectations

Problems also abound in the middle and upper grades. Fractions are poorly developed in the middle grade standards. In sixth grade, students are required to multiply decimals, but fraction multiplication is not introduced until seventh grade. Since decimals are fractions, it is possible that students following these standards will have little if any conceptual understanding of what they are doing when they multiply decimals.

The grade 11 standards expect little from students. We list here all of the algebra standards:

1. *Students use algebraic concepts, symbols, and skills to represent and solve real-world problems.*
2. *Students write, model, and evaluate expressions, functions, equations, and inequalities.*

3. *Students graph linear equations and interpret the results in solving algebraic problems.*

4. *Students solve, graph, or interpret systems of linear equations.*

5. *Students connect algebra with other mathematical topics.*

Important topics are missing from these standards. For example, there are no specific expectations regarding polynomials, linear inequalities, systematic algebraic manipulations, exponential, logarithmic, or trigonometric functions.

Methods and Procedures

Each state's standards documents were evaluated by David Klein, principal author of this report, and at least one other mathematician. Five served as readers, each of whom cooperated with Klein on a different group of states. The readers were: Bastiaan J. Braams, Thomas Parker, William Quirk, Wilfried Schmid, and W. Stephen Wilson.[8] (For biographical information on each, see "About the Expert Panel" on page 127.) The authors of Fordham I and II, Ralph Raimi and Lawrence Braden, served as advisors helping with interpretations of the criteria, providing useful background information, and sharing relevant experiences in producing the previous Fordham reports. Raimi also generously contributed his time to the editing of the introductory material of this report. However, neither Raimi nor Braden served as readers of state standards, and they did not participate in the scoring. We refer to the five readers, together with Raimi and Braden, as the Expert Panel.

At the start of this study, staff of the Thomas B. Fordham Foundation obtained current standards documents and made those available to the Expert Panel. Fordham staff searched state websites for standards documents available for public review. (Among the most positive developments in standards-based reform is the widespread availability of state academic standards documents on the Internet.) Fordham staff also contacted state departments of education (sometimes several times) to confirm that documents available on the web represented the extent of state standards documents. In each case, we received confirmation from state officials that the documents being reviewed represented the full array of standards documents distributed to district and local officials. In cases where the proper documents to be reviewed were in doubt, the lead author of this report, in consultation with the Expert Panel, made the determination based on the following principles: 1. Are the documents readily available or distributed to teachers? 2. Are they meant to guide instruction and not simply test preparation or assessment? 3. Do the documents outline a curriculum or course of study or are they simply guides for pedagogy? To account for the rapid change in state standards over the past six months as this report was being produced, we also periodically checked state standards websites to ensure that the documents under review had not changed. In general, the documents reviewed in this report are current as of September 15, 2004, though in some cases they are even more current.

To calibrate scoring at the beginning of this project, Klein and the five readers each evaluated the standards documents for three states: California, Kansas, and Nebraska. Following extensive discussions related to the criteria for evaluation of these states, Klein and each reader contributed scores for each of these three states. Raimi and Braden also participated in these discussions, helping to ensure consistency of application of the criteria of evaluation between their earlier Fordham reports and this one. After detailed discussions of the standards for these states by the Expert Panel, the differences in scores of the six evaluators were in close accord. The scores for those three states given in this report are averages of the scores of all six evaluators. These are the only states whose rankings were obtained by averaging the scores of all six judges; the evaluations served as standards or models for judging the others.

At this point, each reader was assigned a subset of the remaining 47 states to evaluate with Klein. For the most part, the other states' scores are averages of two readers. Each reader sent notes or a draft report for each of the states on their lists, along with provisional scores, to Klein, who then sent back his own scores and a draft report to the reader for that state. The scores were generally in close agreement, but in those rare instances where there was significant divergence initially, discussions, sometimes lengthy, were necessary to produce agreement on the scores herein reported. In a few cases,

[8] Wilfried Schmid played an important role in the creation of the Massachusetts math standards, and thus did not serve as second reader for that state. For similar reasons, Thomas Parker did not review Michigan's math standards.

other members of the Expert Panel were also consulted. Once agreement on scores and the report for a given state was reached by Klein and the reader for that state, the report was forwarded to the entire Expert Panel for further comments, suggestions, or comparisons.

The criteria for evaluation of state standards used in this report, and described in the section “Criteria for Evaluation,” are the same as those used in Fordham I and II, but the weighting is different. As noted earlier, our *content* criterion scores constitute 40 percent of the total score for each state, compared to 25 percent in Fordham I and II. Since each of the four categories save *reason* has more than one subcategory, there are nine scores (of 0 to 4 each) in all, but when grouped and averaged within each of the main categories we obtained four major scores. These produce an overall score for the state by doubling the resulting *content* score, adding it to the (averaged) scores for *clarity*, *reason*, and *negative qualities*, and dividing the result by five.

The grading scale used in the Fordham II report was retained for this evaluation: 3.25 to 4.0 is an A, indicating excellent performance; 2.5 to 3.24 is a B, indicating good performance; 1.75 to 2.49 is a C, indicating mediocre performance; 1.0 to 1.74 is a D, indicating poor performance; 0.0 to 0.99 is an F, indicating failing performance.

Appendix

Figure 12

STATE	Clearness	Definiteness	Testability	Clarity	Primary	Middle	Secondary	Content	Reason	False Doctrine	Inflation	Negative Qualities	Weighted Final Average	GRADE	2000 Grade	1998 Grade
AL	3.00	3.00	3.00	3.00	3.00	3.00	3.50	3.17	2.00	3.00	4.00	3.50	2.97	B	B	B
AK	1.50	2.50	2.00	2.00	1.50	1.00	1.00	1.17	0.50	1.50	2.00	1.75	1.32	D	D	C
AZ	2.00	2.00	2.00	2.00	3.00	2.00	1.00	2.00	2.00	3.00	1.00	2.00	2.00	C	B	B
AR	2.00	1.50	1.00	1.50	1.00	1.00	0.00	0.67	0.00	0.00	1.50	0.75	0.72	F	D	F
CA	4.00	4.00	3.50	3.83	4.00	4.00	3.83	3.94	3.83	4.00	3.83	3.92	3.89	A	A	A
CO	1.00	1.00	1.00	1.00	2.50	2.50	0.00	1.67	1.00	2.00	1.00	1.50	1.37	D	D	D
CT	2.00	0.00	0.00	0.67	1.00	0.00	0.00	0.33	0.00	0.00	2.00	1.00	0.47	F	D	D
DE	2.00	0.50	0.00	0.83	1.00	1.00	0.00	0.67	0.50	0.00	0.00	0.00	0.53	F	C	C
DC	2.00	1.50	1.50	1.67	1.00	1.00	2.00	1.33	1.50	0.00	2.00	1.00	1.37	D	B	D
FL	2.00	1.00	1.00	1.33	1.00	1.00	0.00	0.67	1.50	0.00	1.00	0.50	0.93	F	D	D
GA	4.00	3.00	3.00	3.33	2.00	3.00	3.00	2.67	2.00	2.00	2.00	2.00	2.53	B	B	B
HI	1.00	1.00	1.00	1.00	0.50	0.50	0.00	0.33	0.00	0.00	1.00	0.50	0.43	F	C	F
ID	2.50	1.00	1.50	1.67	1.50	0.50	0.00	0.67	1.00	1.00	2.00	1.50	1.10	D	-	F
IL	2.00	1.50	1.00	1.50	1.50	2.00	2.50	2.00	1.00	2.00	3.00	2.50	1.80	C	D	D
IN	3.00	4.00	4.00	3.67	3.50	4.00	4.00	3.83	4.00	4.00	3.50	3.75	3.82	A	C	C
IA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KS	2.00	1.50	1.50	1.67	1.50	1.00	0.33	0.94	0.33	0.17	0.33	0.25	0.83	F	A	D
KY	1.50	2.00	2.00	1.83	2.00	3.00	2.00	2.33	1.00	1.50	1.50	1.50	1.80	C	B	D
LA	2.00	2.00	2.00	2.00	2.50	2.00	2.50	2.33	1.00	0.50	2.00	1.25	1.78	C	F	F
ME	2.00	0.50	1.00	1.17	1.50	1.00	1.00	1.17	0.50	2.00	3.50	2.75	1.35	D	D	F
MD	2.00	2.00	2.00	2.00	1.50	2.50	1.00	1.67	1.50	2.00	2.00	2.00	1.77	C	C	F
MA	4.00	3.50	3.50	3.67	4.00	4.00	3.00	3.67	2.00	4.00	3.00	3.50	3.30	A	D	F
MI	2.50	2.00	2.00	2.17	2.00	3.00	0.00	1.67	2.00	2.00	3.00	2.50	2.00	C	F	F
MN	2.00	2.00	2.00	2.00	1.00	1.00	3.00	1.67	1.00	2.00	2.00	2.00	1.67	D	F	-
MS	2.00	1.00	1.00	1.33	1.50	2.00	2.50	2.00	1.00	1.00	3.00	2.00	1.67	D	A	B
MO	1.00	0.00	1.00	0.67	1.00	0.00	0.00	0.33	1.00	1.00	0.00	0.50	0.57	F	F	F
MT	2.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	2.00	2.00	2.00	1.00	D	D	F
NE	1.67	1.50	2.00	1.72	1.83	1.17	0.83	1.28	0.67	1.50	2.83	2.17	1.42	D	C	F
NV	3.00	1.50	2.00	2.17	2.00	1.00	1.00	1.33	1.50	2.00	3.00	2.50	1.77	C	C	-
NH	1.50	1.00	1.00	1.17	1.00	1.00	0.00	0.67	0.00	1.00	1.00	1.00	0.70	F	C	C
NJ	3.00	2.00	1.50	2.17	1.50	1.00	1.00	1.17	0.50	0.00	1.50	0.75	1.15	D	C	C

STATE	Clearness	Definiteness	Testability	Clarity	Primary	Middle	Secondary	Content	Reason	False Doctrine	Inflation	Negative Qualities	Weighted Final Average	GRADE	2000 Grade	1998 Grade
NM	3.00	3.00	3.00	3.00	3.00	2.00	3.00	2.67	2.00	3.00	3.00	3.00	2.67	B	F	F
NY	1.00	1.50	2.00	1.50	2.00	2.00	3.00	2.33	2.00	2.00	2.50	2.25	2.08	C	B	B
NC	3.00	2.00	2.00	2.33	1.00	2.00	1.50	1.50	1.50	1.50	3.00	2.25	1.82	C	A	A
ND	3.00	2.00	2.00	2.33	3.00	1.00	0.00	1.33	1.00	2.00	4.00	3.00	1.80	C	D	D
OH	2.00	2.00	2.00	2.00	1.00	1.00	2.00	1.33	1.00	1.00	2.00	1.50	1.43	D	A	A
OK	2.50	2.00	2.00	2.17	1.50	2.00	2.00	1.83	1.50	2.00	3.00	2.50	1.97	C	B	F
OR	3.00	2.00	2.50	2.50	1.00	1.00	1.00	1.00	0.00	2.00	2.50	2.25	1.35	D	D	D
PA	2.00	1.00	1.00	1.33	2.00	1.00	0.50	1.17	1.00	2.00	1.50	1.75	1.28	D	C	D
RI	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.67	0.00	1.00	1.00	1.00	0.67	F	F	F
SC	1.00	1.00	1.00	1.00	1.50	1.50	2.00	1.67	1.50	0.50	1.00	0.75	1.32	D	B	D
SD	3.00	1.50	2.00	2.17	2.00	1.00	2.00	1.67	1.00	2.00	3.00	2.50	1.80	C	A	F
TN	3.00	1.00	1.50	1.83	1.00	2.00	1.00	1.33	2.00	1.00	3.00	2.00	1.70	D	F	C
TX	3.00	3.00	2.00	2.67	2.00	1.00	2.00	1.67	1.00	2.00	2.00	2.00	1.80	C	B	B
UT	3.00	1.50	1.00	1.83	0.50	1.50	1.50	1.17	0.50	0.50	1.50	1.00	1.13	D	B	B
VT	1.00	1.33	1.67	1.33	1.33	1.00	0.67	1.00	0.67	2.00	2.00	2.00	1.20	D	C	C
VA	3.00	3.00	2.50	2.83	2.00	2.00	2.00	2.00	1.50	1.00	2.00	1.50	1.97	C	B	B
WA	1.00	0.00	0.00	0.33	1.00	2.00	0.00	1.00	0.50	0.00	0.00	0.00	0.57	F	F	F
WV	2.00	2.00	2.00	2.00	2.00	3.00	2.50	2.50	3.00	1.50	2.00	1.75	2.35	C	B	B
WI	3.00	1.00	1.00	1.67	2.00	2.00	1.00	1.67	1.00	1.00	2.00	1.50	1.50	D	C	C
WY	2.00	0.00	1.00	1.00	1.00	1.00	0.50	0.83	0.00	1.50	3.00	2.25	0.98	F	D	-

(A = 4.00 - 3.25; B = 3.24 - 2.50; C = 2.49 - 1.75; D = 1.74 - 1.00; F = 0.99 - 0.00)

Figure 13: National GPA Trend

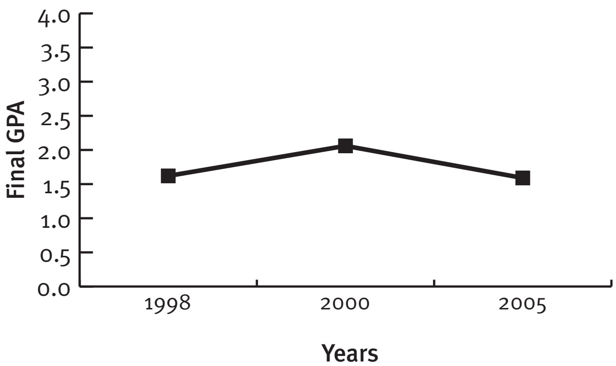


Figure 16: Clarity

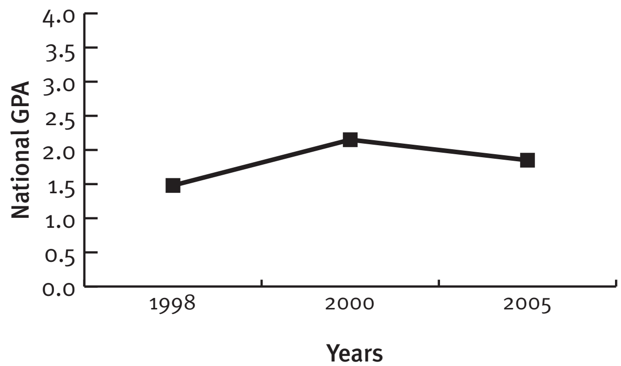


Figure 14: Final Grade Distribution 1998 - 2005

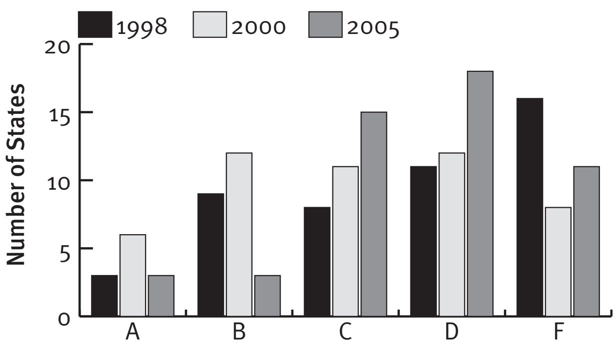


Figure 17: Grade Distribution for Content, 1998 - 2005

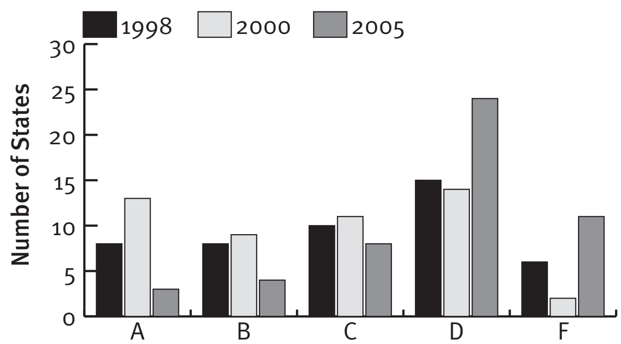


Figure 15: Grade Distribution for Clarity, 1998 - 2005

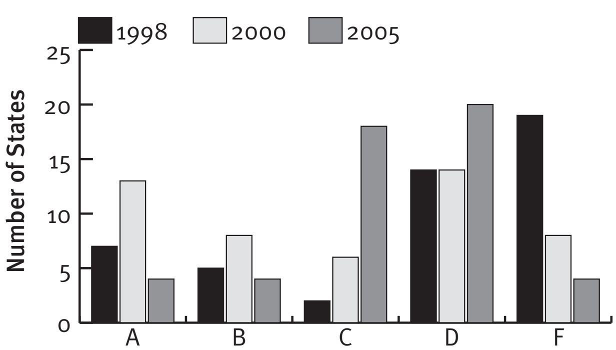


Figure 18: Content

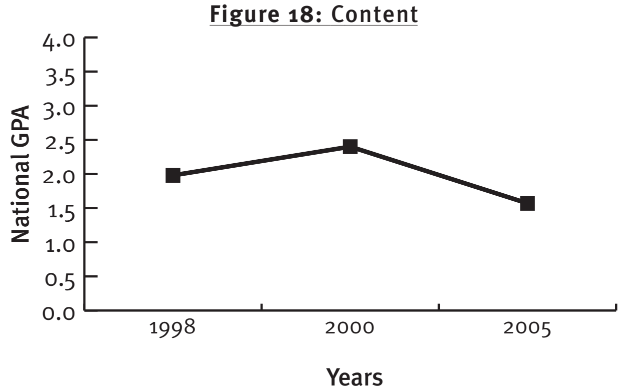


Figure 19: Grade Distribution for Reason, 1998 - 2005

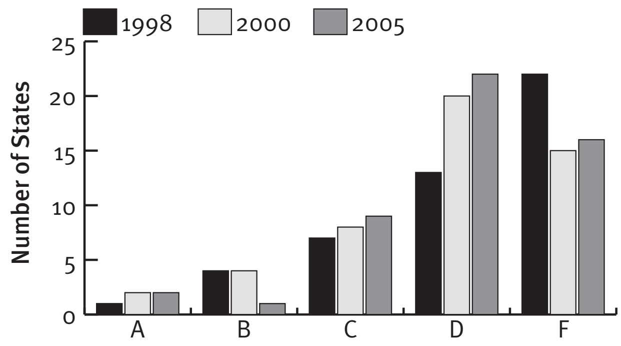


Figure 22: Negative Qualities

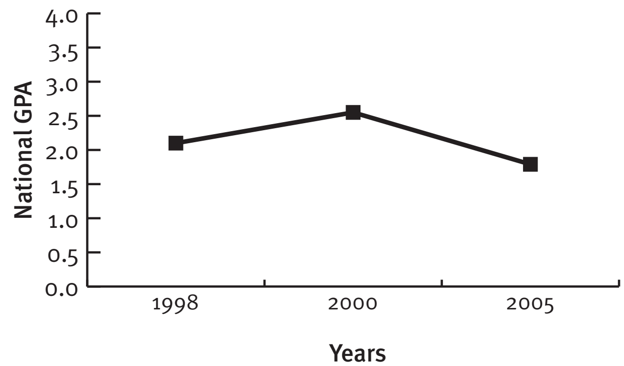


Figure 20: Reason

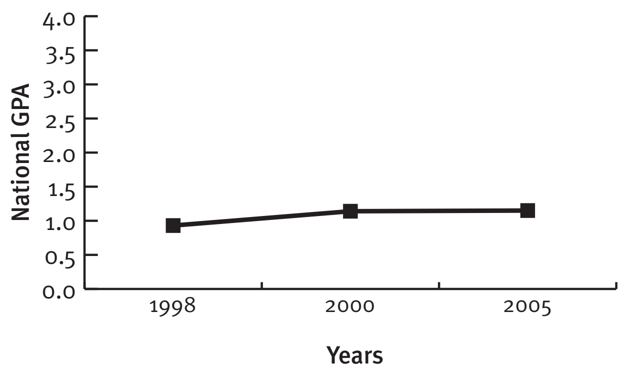
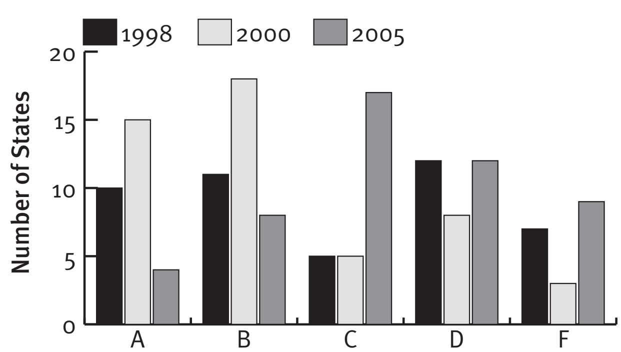


Figure 21

Grade Distribution for Negative Qualities, 1998 - 2005



About the Expert Panel

David Klein is a Professor of Mathematics at California State University, Northridge. He received a B.A. in mathematics and a B.S. in physics from the University of California at Santa Barbara, and a Ph.D. in applied mathematics from Cornell University, and has held teaching and research positions at Louisiana State University, UCLA, and USC. He has published research papers in mathematical physics and probability theory as well as articles about K-12 education. Professor Klein has testified about mathematics education in forums ranging from local school boards to a subcommittee of the U.S. House of Representatives. He has served on official panels to review K-8 mathematics curriculum submissions for statewide adoption in California. In 1999 he was appointed by the California State Board of Education to review and evaluate professional development proposals for California mathematics teachers. From 1999 to 2000, he served as Mathematics Content Director for the Los Angeles County Office of Education, where he directed and assisted math specialists.

Bastiaan J. Braams is Visiting Professor in the department of mathematics and computer science at Emory University in Atlanta. His degrees are in physics (Ph.D., Utrecht, the Netherlands, 1986) and his research work is in computational science, and especially in fusion energy, quantum chemistry, and molecular physics.

Lawrence Braden has taught elementary, junior high, and high school mathematics and science in Hawaii, Russia, and at St. Paul's School in New Hampshire. He served as co-author of the previous Fordham Foundation evaluations of state math standards in 1998 and 2000. In 1987 he received the Presidential Teaching Award for Excellence in Teaching Mathematics.

Thomas Parker is Professor of Mathematics at Michigan State University. He received his Ph.D. from Brown University in 1980. His research is in geometric analysis and its connections with mathematical physics. He teaches "Mathematics for Elementary School Teachers," a course for which he has also co-authored a textbook.

William Quirk holds a Ph.D. in mathematics from New Mexico State University. After teaching university-level math and computer science for eight years, he embarked on a 20-year career teaching interactive systems design. In 1996, Quirk began a public service endeavor to help parents understand the constructivist approach to math education. His essays can be found at wgquirk.com.

Ralph A. Raimi is Professor Emeritus of Mathematics at the University of Rochester. He received his Ph.D. from the University of Michigan and was a Fulbright Fellow. Dr. Raimi has served as a consultant to states on K-12 mathematics education and was lead author of the 1998 and 2000 Fordham Evaluations of state mathematics standards.

Wilfried Schmid is Professor of Mathematics at Harvard University. Schmid grew up in Bonn, Germany, and received his Ph.D. from the University of California at Berkeley in 1967. He has served as Mathematics Advisor to the Massachusetts Department of Education, as a member of the Steering Committee of the National Assessment of Educational Progress (NAEP), and as member of the Program Committee of the International Congress of Mathematics Education, 2004.

W. Stephen Wilson was raised in Kansas and educated at MIT, and has taught at Johns Hopkins University for more than 25 years. His research focus is algebraic topology. His work in K-12 mathematics education has been mostly with parent advocacy groups and on state mathematics standards. Wilson was recently appointed to the Johns Hopkins Council on K-12 Education.

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